OPTICAL COMPUTATION OF THE LAPLACE OPERATOR AT NORMAL INCIDENCE USING A MULTILAYER METAL-DIELECTRIC STRUCTURE

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Abstract. We theoretically and numerically investigate the optical implementation of the second-order spatial differentiation operation using a layered metal-dielectric structure at normal light beam incidence. Numerical simulation results confirm the theoretical results and show the possibility of optical calculation of the Laplace operator with high quality.

Keywords: *optical differentiation, second derivative, Laplace operator, layered structure, normal incidence, resonance*

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INTRODUCTION

The possibility of optical computation of the Laplace operator is of great interest for optical information processing tasks, especially analog optical computing and image processing tasks [1-7]. The Laplace operator can be applied to detect changes in brightness or contrast in images, as it allows to highlight object contours, line intersection points and other important details, which, in particular, can improve the accuracy of image classification [8].

The optical realization of the Laplace operator requires diffraction structures possessing a second-order zero in spatial frequencies in the reflection or transmission spectrum. This condition is easiest to fulfill in the normal incidence geometry, since in this case, due to the symmetry of the diffraction problem, the reflection (transmission) zeros will be second-order zeros [1]. In [1, 3, 9, 10], layered diffraction structures (in reflection) [1, 9] and diffraction gratings with two-dimensional periodicity (in transmission) [3, 5, 10] were successfully used for the optical realization of second-order differentiation operators. Note that the previously considered layered dielectric structures (Bragg gratings with period defect) for optical calculation of the Laplace operator [1] are simpler in terms of their fabrication compared to diffraction gratings. At the same time, they have a horizontal plane of symmetry, which creates difficulties in their practical realization (because, in particular, the location of these structures on the substrate breaks the symmetry and leads to the disappearance of the reflection zero).

According to the authors of the present work, metallodielectric layered structures, in comparison with purely dielectric Bragg structures, have an advantage in calculating the Laplace operator at normal incidence because they do not require the presence of a horizontal plane of symmetry.

In the present work, a layered metal-dielectric-metal structure "metal-dielectric-metal-dielectric" located on a metal substrate is investigated. It is shown that such a structure allows to perform the optical calculation of the Laplace operator from the incident optical beam profile with high accuracy. At the same time, the numerical simulation results are in full agreement with the theoretical description.

OPTICAL BEAM TRANSFORMATION AT REFLECTION FROM THE STRUCTURE

Let us first consider the transformation of the *x-component of* the electric field of a three-dimensional linearly polarized optical beam occurring during its reflection from some layered structure at normal incidence. Following [1], we can show that the *x-component of* the electric field of the reflected beam, represented as a plane wave expansion, will have the following form

$$E_{\mathrm{refl},x}(x,y) = \iint G_x(k_x,k_y) H(k_x,k_y) \exp\left(\mathrm{i}k_x x + \mathrm{i}k_y y\right) \mathrm{d}k_x \mathrm{d}k_y, \quad (1)$$

where $G_x(k_x,k_y)$ is the spectrum of *the x-component of* the electric field of the incident beam, representing the amplitudes of plane waves with tangential components of the wave vectors (spatial frequencies) k_x, k_y, forming the incident beam, and $H(k_x,k_y)$ *is the* transfer function (TF), which describes the transformation of the spectrum of the incident beam (change in the amplitudes of plane waves occurring during reflection). As shown in [1], this TF is expressed through the reflection coefficients of the structure for TE- and TM-polarized plane waves and for a layered structure with zero reflection at normal incidence has the following Taylor series expansion to quadratic terms in the vicinity of zero:

$$H(k_x, k_y) \approx c_{x,2} k_x^2 + c_{y,2} k_y^2 . \quad (2)$$

Thus, in the neighborhood of zero, the PF contains only quadratic terms. In this case, as follows from formulas (1), (2), the structure realizes the following operation of second-order differentiation of the transverse profile of the *x-component of* the incident beam electric field:

$$E_{\mathrm{refl},x}(x, y) = -c_{x,2} \frac{\partial^2 E_{\mathrm{inc},x}(x, y)}{\partial x^2} - c_{y,2} \frac{\partial^2 E_{\mathrm{inc},x}(x, y)}{\partial y^2} . \quad (3)$$

Obviously, if the coefficients $c_{(x)\ (,2)}$and $c_{(y)\ (,2)}$in (2) are equal, the reflected beam profile (3) will be proportional to the Laplace operator from the profile of the incident beam. As noted above, this case is of most practical interest and will therefore be discussed below.

GEOMETRY OF THE INVESTIGATED METAL-DIELECTRIC LAYERED STRUCTURE AND OBTAINING ZERO REFLECTION

To calculate the second-order differential operator (3) at normal incidence, we propose to use a four-layer metal-dielectric-metal-dielectric (MDMD) structure located on a substrate (optically thick layer) made of metal. We will assume that above the structure there is a medium with refractive index n_{sup}= 1 (Fig. 1).

In [11, 12], a method for calculating three-layer metal-dielectric structures consisting of two metal layers separated by a dielectric layer and having zero reflection was considered. We use an approach similar to these works to calculate the parameters of the investigated four-layer MDMD structure having zero reflection. The formula for calculating the thickness of the upper metal layer at the given thicknesses of the "lower" pair of metal and dielectric layers to achieve zero reflection [11] is as follows:

$$\left|\frac{r}{r^2 - t^2}\right| = |\rho|, \tag{4}$$

where r, t are the complex reflection and transmission coefficients of the upper metallic layer considered as functions of its thickness h_1; ρ is the complex reflection coefficient of the "lower" pair of layers consisting of metallic layer with thickness $h_3$and dielectric layer with thickness $h_{(4)}$ (Fig. 1). Note that in formula (4) the coefficients r and t are assumed to be calculated at fixed wavelength λ and polarization of the incident wave. After finding the thickness of the first (upper) metallic layer $h_{(1)}$,the thickness of the dielectric layer $h_{(2)}$ of the "upper" pair, providing zero reflection, can be found from the formula [11]

$$h_2 = \frac{1}{2k_0 n}\left[\arg\frac{r}{r^2 - t^2} - \arg\rho + 2\pi j\right], \tag{5}$$

where $k_0$is the wave number, n is the refractive index of the dielectric layer, j *is* an integer providing a positive value of the thickness h_2.

It should be noted that the structure whose parameters are calculated by formulas (4) and (5) in the case of normal incidence will have zero second-order reflection at spatial frequencies due to the symmetry of the diffraction problem.

NUMERICAL SIMULATION RESULTS

Let us now consider the possibility of using the investigated four-layer MDMD-structure to calculate the Laplace operator from the profile of the *x-component of* the electric field of the incident beam. According to (2), (3), for this purpose it is necessary to fulfill the condition $c_{(x)\,(,2)} = c_{(y)\,(,2)}$. The possibility of achieving the above condition was investigated for MDMD-structures in the configuration "Cu-TiO_2-Cu-$TiO_{(2)}$ "(materials of layers - copper and titanium dioxide) on a substrate of chromium (Cr) at a fixed wavelength λ = 633 nm and TM- polarization. The optimization parameters were the thicknesses of the lower "metal-dielectric pair" layers $h_3$and $h_{(4)}$ (Fig. 1), and the thicknesses of the upper two layers $h_1$and $h_2$were calculated using formulas (4) and (5) from the condition of obtaining zero reflection. For the selected wavelength, the following values of refractive indices for the above materials were used [13, 14]: n_{Cu}= 0.23+ 3.*43i* (Cu), n_{TiO2}= 2.58 (TiO_2), n_{Cr}= 3.14+ 3.*31i* (Cr).

As a result of calculations, a structure with the following layer thicknesses was found: h_1= 7.2 nm, h_2= 51 nm, h_3= 34.0 nm, h_4= 79.1 nm. For this structure, the condition $c_{x,2}= c_{y,2}$is satisfied with high accuracy $c_{y,2} = 0.034 \cdot e^{-2.92i}$ μm$^{(2)}$.

Fig. *2a* shows the PF modulus of the investigated structure calculated numerically within the framework of the strict solution of Maxwell's equations by the method [15]. The PF modulus in Fig. *2a* turned out to be visually indistinguishable from the modulus of the "model" transfer function (2) at the values of the coefficients given above, so the latter is not shown for brevity. The deviation between the moduli of the model function and the numerically calculated PF is small: the RMS normalized to the maximum of the PF modulus is only 0.73%, and the maximum deviation is 1.94%. Note that the strictly calculated PF has the required quadratic form at

$\sqrt{k_x^2 + k_y^2} / k_0 \leq 0.135$, which corresponds to a spatial resolution of ~3.7λ [10] . The achieved spatial resolution practically coincides with the spatial resolution obtained in [10], where a much more complicated structure in the form of metasurface with essentially subwavelength dimensions of the unit cell details was used for the calculation of the Laplace operator.

Next, we consider the transformation of the x-component *of* the electric field occurring at reflection from the investigated structure of an incident Gaussian beam:

$$E_{\mathrm{inc},x}(x,y) = \exp\left[-\frac{x^2+y^2}{\sigma^2}\right]. \tag{6}$$

For the case under consideration, the "model" function describing the profile of the reflected beam and calculated by formula (3) will have the following form:

$$E_{\mathrm{refl},x}(x,y) = -4\frac{c_{x,2}}{\sigma^4}\left(x^2+y^2-\sigma^2\right)\exp\left[-\frac{x^2+y^2}{\sigma^2}\right]. \tag{7}$$

Fig. *2b* shows the modulus of the rigorously calculated reflected beam profile at σ = 6 μm. As in the case of the transfer function, the model (7) and the strictly calculated reflected beam profiles match well, with a normalized RMS error of only 0.45% and a maximum deviation of 1.07%.

As mentioned in the introduction, the Laplace operator is widely used to extract contours (brightness variations) in an image. As an example illustrating this operation, consider an incident beam with a profile of the *x-component of* the electric field in the form of a so-called super Gaussian function

$$E_{\mathrm{inc},x}(x,y) = \exp\left[-\frac{x^6+y^6}{\sigma^6}\right]. \tag{8}$$

Fig. *3a* shows the result of MDMD-structure conversion of the incident beam with profile (8) at σ= 3 μm, and Fig. *3b* - normalized cross sections of the incident and reflected beam profiles at y = 0. Fig. 3 shows the appearance of double contours characteristic of the Laplace operator at the boundaries of the input beam with a quasi-rectangular shape and a width on the recession level of 0.5 in 6 μm.

CONCLUSION

Thus, we have investigated the optical realization of the second-order spatial differentiation operation at normal incidence of the optical beam using a layered metal-dielectric structure. The parameters of the four-layer MDMD structure are found, under which the condition required for the optical calculation of the Laplace operator from the incident beam profile is fulfilled. Numerical simulation results confirm the theoretical results and show the possibility of optical calculation of the Laplace operator with high quality (with RMS error less than 1%). The results obtained can be applied in the development of analog optical computing and optical information processing systems.

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FIGURE CAPTIONS

Fig. 1. Geometry and parameters of the investigated metal-dielectric layered structure.

Fig. 2. Modulus of the strictly calculated PF of the metallodielectric layered structure performing the calculation of the Laplace operator (a) and the absolute value normalized to the maximum value of *the* x-component *of* the electric field of the numerically calculated reflected beam (b).

Fig. 3. Maximal value-normalized profile modulus *of* the x-component *of* the reflected electric field formed at the incident beam with a profile in the form of a supergaussian function (a); Cross sections of the maximal value-normalized profiles of the reflected beam $E_{(\mathrm{refl})\ (,)}(_{x)(}x, y)_{(}$black line) and the incident beam $E_{(\mathrm{inc})\ (,)\ (x)(}x, y)$ along the *x-axis* at $y = 0$ (red dashed line) (b). The blue arrow shows the width of the incident beam at level 0.5.

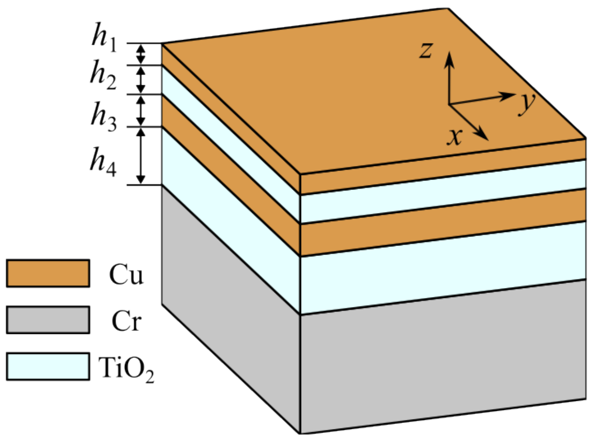

Fig. 1.

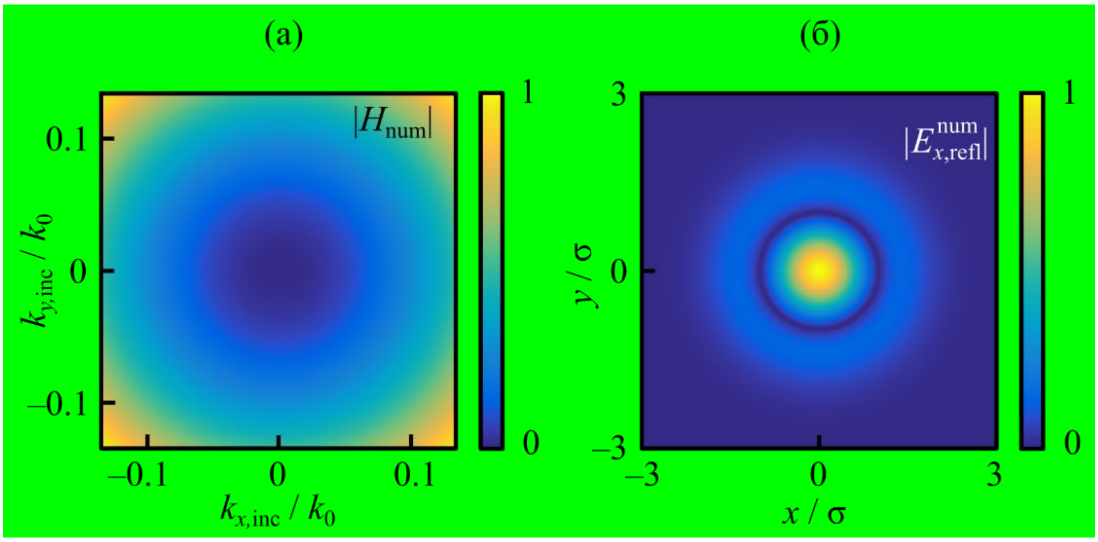
(а)
(б)
$|H_{num}|$
$|E^{num}_{x,refl}|$
$k_{y,inc} / k_0$
$k_{x,inc} / k_0$
y / σ
x / σ

Fig. 2.

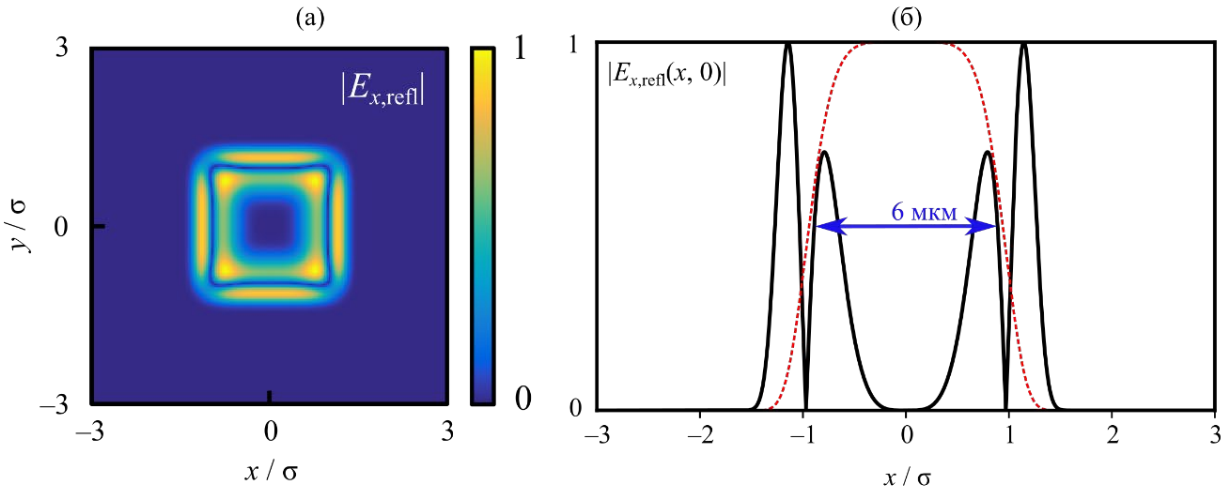
(а)
(б)
$|E_{x,refl}|$
$|E_{x,refl}(x, 0)|$
6 мкм
y / σ
x / σ

Fig. 3.