PLASMA WAVES IN A TWO-DIMENSIONAL SUPERLATTICE UNDER THE INFLUENCE OF A NONLINEAR ELECTROMAGNETIC WAVE

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Abstract. An expression is obtained that allows us to determine the law of dispersion of plasma waves $\omega(k)$ in a two-dimensional semiconductor superlattice under the influence of a nonlinear electromagnetic wave and, in the extreme case of weak nonlinearity, an analytical expression for $\omega(k)$ is found. The possibility of controlling the frequency of the plasma wave by the parameters of a nonlinear wave has been established.

Keywords: *semiconductor superlattice, two-dimensional electron gas, energy spectrum, nonlinear electromagnetic wave, plasma waves*

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INTRODUCTION

The quantum theory of plasma waves for semiconductor superlattices (SRs) was constructed in [1,2]. An important direction of the study of collective phenomena in condensed media is the study of plasma wave propagation in low-dimensional electronic systems and, in particular, in two-dimensional (2D) semiconductor SRs. The attention of researchers to the study of nonlinear electromagnetic waves propagating in semiconductor structures with a nonquadratic dispersion law is not weakening. In [3-5], nonlinear plasma oscillations of electron gas in semiconductor quantum SR were studied. In the regime of rare collisions ($\nu << \omega_p$, the collision frequency of the electron with irregularities of the crystal lattice ν is much smaller than the generalized plasma frequency of the electron in the minizone ω_p), the propagation of the electromagnetic wave along the SR layers, when its field is directed along the SR axis, is described by the Sine-Gordon equation (SGE) [6]. One of the most general periodic solutions of the SGE are those expressed through elliptic Jacobi functions and called knoidal waves. When the characteristic distance at which a noticeable change in the wave field occurs is much larger than the length of the electrons' free path, the wave field can be considered homogeneous. In this case, the electric field strength of the wave has the form [7-9]

$$E_x(t) = E_0 cn\left(\frac{2K(\kappa)\omega_0}{\pi}t, \kappa\right), \tag{1}$$

where $cn(x)$ is the elliptic Jacobi function, $\kappa = \frac{eE_0 d}{2\omega_p} \cdot \frac{\sqrt{\beta^2 - 1}}{\beta}$ is the nonlinearity modulus (hereinafter $\hbar = 1$), , $\omega_0 = \frac{\pi}{2K(\kappa)} \frac{\beta}{\sqrt{\beta^2 - 1}} \omega_p$ $\beta =$ u/V ($\beta > 1$), V is the wave velocity in the absence of electrons, u is the phase velocity of the wave, $K(k)$ is the full elliptic integral of the first kind, E_0 is the amplitude of the nonlinear wave field strength. In [7-9], the peculiarities of the influence of the nonlinear knoidal wave field on various physical processes in SR are considered. Recently, a significant number of works devoted to the theory of knoidal waves have appeared [10-13]. This emphasizes the relevance of the present study.

The processes of plasma wave propagation in 2D semiconductor SRs are of great interest in semiconductor physics. The possibility of plasma waves and the density of plasma excitations in 2D electron gas SRs have been studied in [14, 15]. In [16, 17], the effect of strong DC and AC electric field on plasma oscillations in 2D electron gas SRs was investigated. The possibility of propagation of solitary electromagnetic waves in 2D SR is shown in [18]. Works [19-23] are devoted to the study of new graphene-based materials - graphene SRs and peculiarities of the law of dispersion of plasma waves in such structures. In the case of weak non-additivity of the energy spectrum of graphene SRs, the expressions for finding the law of dispersion of plasma waves $\omega(k)$ obtained for quantum semiconductor SRs can be used to estimate $\omega(k)$ both in the absence of external influences and in strong static and alternating electric fields.

This paper investigates the effect of a nonlinear electromagnetic wave on the dispersion law of plasma waves in a two-dimensional semiconductor quantum superlattice.

MAIN PART

The energy spectrum of charge carriers in 2D SR can be chosen in model form [14-15]

$$\varepsilon(\vec{p}) = \Delta - \frac{\Delta}{2}\left[\cos(p_x d) + \cos(p_y d)\right], \tag{2}$$

whereΔ is the half-width of the conduction minizone; d *is* the SR period; $\vec{p}$ is the electron quasi-impulse. We limit ourselves to the one-minizone approximation, neglecting interminizone transitions.

Fig. 1 shows the geometry of the problem. Let a nonlinear electric field (1) be applied in the direction of the OX axis of SR, which will be described by the vector potential $\vec{A}(t)=\{-c\Phi(t)/ed,\ 0\}$. It is more convenient to pass from the nonlinear wave intensity to the dimensionless potential *F(t)*

$$\Phi(t)=2\arcsin\left\{\kappa\, sn\left(\frac{2K(\kappa)\,\omega_0}{\pi}t,\,\kappa\right)\right\}, \tag{3}$$

where $0<k\leq 1$, $sn(x)$ is an elliptic Jacobi function.

In the self-consistent field approximation, the Hamiltonian of interacting electrons taking into account the overshooting processes has the form [16,17]

$$H=\sum_{\vec{p}}\varepsilon\left(\vec{p}+\frac{e}{c}\vec{A}(t)\right)a^{+}_{\vec{p}}a_{\vec{p}}+e\frac{1}{\sqrt{N_xN_y}}\sum_{\vec{p},\vec{k}}\sum_{n,m}U\left(\vec{k},t\right)\times$$

$$\times M(k_x)M(k_y)\,a^{+}_{\vec{p}-\vec{k}+\vec{g}}\,a_{\vec{p}}\,, \tag{4}$$

where $a^{+}_{\vec{p}}$, $a_{\vec{p}}$ are the birth and annihilation operators of an electron with momentum $\vec{p}$ *;* $N_x$and $N_y$are the number of potential wells forming the SR along the x and y axes, respectively, , $\vec{g}=(n\,{}^{2\pi}\!/_{d}, m\,{}^{2\pi}\!/_{d})$

$$M(k_x)=\int_0^{N_xd}\varphi^*(x)\varphi(x)\exp(-ik_xx)dx\,, \tag{5}$$

$$M(k_y)=\int_0^{N_yd}\varphi^*(y)\varphi(y)\exp(-ik_yy)dy\ ,$$

$U(\vec{k},t)$ - self-consistent potential defined by the following relation

$$U(\vec{k},t)=\frac{2\pi e}{\chi k}\sum_{\vec{p}}\sum_{n,m} < a^{+}_{\vec{p}+\vec{k}+\vec{g}}\, a_{\vec{p}} > \; M(-k_x)M(-k_y), \tag{6}$$

χ - is the dielectric constant of the lattice.

A good enough approximation, which is used in the theoretical physics of low-dimensional systems, is the random phase approximation [14-17, 19, 21-23]. The equation of motion for the mean $< a^{+}_{\vec{p}+\vec{k}+\vec{g}}\, a_{\vec{p}} >$ in this case has the form [16, 17]

$$\left\{\frac{\partial}{\partial t}+i\left[\varepsilon\left(\vec{p}+\vec{k}+\frac{e}{c}\vec{A}(t)\right)-\varepsilon\left(\vec{p}+\frac{e}{c}\vec{A}(t)\right)\right]\right\} < a^{+}_{\vec{p}+\vec{k}+\vec{g}}\, a_{\vec{p}} >=$$

$$-ieU(\vec{k}+\vec{g},t)M\left(\left[\vec{k}+\vec{g}\right]_x\right)M\left(\left[\vec{k}+\vec{g}\right]_y\right)(n_{\vec{p}+\vec{k}+\vec{g}}-n_{\vec{p}}), \tag{7}$$

where $n_{\vec{p}} = < a^{+}_{\vec{p}} a_{\vec{p}} >$ - numbers of filling of electronic levels in 2D - electron gas.

The solution of equation (7) has the form

$$< a^{+}_{\vec{p}+\vec{k}+\vec{g}}\, a_{\vec{p}} >=-ie\int_{-\infty}^{t} dt'\, U(\vec{k}+\vec{g},t)M\left(\left[\vec{k}+\vec{g}\right]_x\right)M\left(\left[\vec{k}+\vec{g}\right]_y\right)\times$$

$$\times(n_{\vec{p}+\vec{k}+\vec{g}}-n_{\vec{p}})\exp\left\{i\int_{t}^{t'}\left[\varepsilon\left(\vec{p}+\vec{k}+\frac{e}{c}\vec{A}(t'')\right)-\varepsilon\left(\vec{p}+\frac{e}{c}\vec{A}(t'')\right)\right]dt''\right\}. \tag{8}$$

To calculate the second integral included in expression (8), we will use the decomposition ofsin $\Phi(t)$ andcos $\Phi(t)$ into a trigonometric Fourier series for the value of the nonlinearity modulus $\kappa \in (0,1)$:

$$\cos\Phi(t)=\sum_{n=0}^{\infty}a_n\cos 2n\omega_0 t,\quad \sin\Phi(t)=\sum_{n=0}^{\infty}b_n\sin(2n+1)\omega_0 t \tag{9}$$

$$a_0=2\frac{E(\kappa)}{K(\kappa)}-1,\;,a_n=4n\frac{\pi^2}{K^2(\kappa)}\frac{q_\kappa^{n}}{1-q_\kappa^{2n}}\; b_n=2(2n+1)\frac{\pi^2}{K^2(\kappa)}\frac{q_\kappa^{n+\frac{1}{2}}}{1+q_\kappa^{2n+1}}\;, \tag{10}$$

$$q_\kappa=\exp\left(-\frac{\pi K'(\kappa)}{K(\kappa)}\right),\;.K'(\kappa)=K\left(\sqrt{1-\kappa^2}\right)$$

Substituting the solution of equation (7) into (6), we obtain the equation defining the law of dispersion of plasma waves $\omega(\vec{k})$. Estimates show that at $\omega_0 > \Delta$ and $k <$ 0.5, only the first terms of the sums (9) can be left in the dispersion equation with a sufficient degree of accuracy. Then the dispersion equation will take the form

$$\frac{2\pi e^2}{\chi}\Pi(\vec{k},\omega)\, S(\vec{k}) = 1, \tag{11}$$

$$S(\vec{k}) = \sum_{n,m} \frac{\left|M([\vec{k}+\vec{g}]_x)\right|^2 \left|M([\vec{k}+\vec{g}]_y)\right|^2}{\sqrt{(k_x+g_x)^2+(k_y+g_y)^2}}, \tag{12}$$

$$\Pi(\vec{k},\omega) = \sum_p J_0^2\left[\Delta\sin(p_x d+\alpha_x)\sin(\alpha_x)\frac{a_1(\kappa)}{2\omega_0}\right] J_0^2\left[\Delta\cos(p_x d+\alpha_x)\sin(\alpha_x)\frac{b_0(\kappa)}{\omega_0}\right]\times$$

$$\times\frac{n(\vec{p}+\vec{k})-n(\vec{p})}{\Delta\left[\sin(p_x d+\alpha_x)\sin(\alpha_x)a_0(\kappa)+\sin(p_y d+\alpha_y)\sin(\alpha_y)\right]-\omega}, \tag{13}$$

where , . $\alpha_x = k_x d/2$ $\alpha_y = k_y d/2$

To obtain the explicit form $S(\vec{k})$ we need to specify the form of potential pits in SR. When $\varphi(x) = const$ at $0 \le x \le d$, and $\varphi(x) = 0$ at $x < 0$, $x >$ d, formula (12) will take the following form

$$S(\vec{k}) = \frac{4}{d^4}\sum_{n,m}\frac{[1-\cos(k_x d)][1-\cos(k_y d)]}{(k_x+g_x)^2(k_y+g_y)^2\sqrt{(k_x+g_x)^2(k_y+g_y)^2}}. \tag{14}$$

Even in such a simple model case, for arbitrary values of $\vec{k}$ one cannot obtain an analytical expression for $S(\vec{k})$. However, at small values of k $\left(k_x, k_y << \pi/d\right) S(\vec{k}) \sim$ *1/k* and the plasmon spectrum has a dispersion $\omega^2 \sim k$, characteristic of a 2D gas without a periodic potential.

For the ungenerated electron gas in the limit of high temperatures:Δ << T we consider a special case for which we can obtain an analytical expression of the dispersion lawω *(k)*. Atω_0 >> Δ and k_y=0 we obtain:

$$\omega(k_x) = a_0(\kappa)\Delta\left|\sin\frac{k_x d}{2}\right|\frac{f(k_x)}{\sqrt{f(k_x)^2-1}}, \qquad (15)$$

Where . $f(k_x) = 1 + \frac{\chi T}{2\pi e^2 N_0}\frac{a_0(\kappa)}{S(k_x)}$

Fig. 2 shows a plot of the dependence of the plasma wave frequency on the wave number$\omega(k)$, obtained by numerical analysis of formula (15). The possibility of controlling the plasma wave frequency by the parameters of the external nonlinear wave is established. It follows from Fig. 2 that a decrease in the amplitude of the nonlinear wave leads to an increase in the plasma frequency. At$kd/2 << 1$ we obtain the expected dispersion dependence$\omega^2 \sim k$, characteristic of a 2D gas without SR. In the limit of linear waves (*to*→ 0), expression (15) corresponds to the law of dispersion of plasma waves in a 2D electron gas SR under the influence of an alternating high-frequency electric field [17]. In the case when E_0= 0 (*to*= 0) the limiting transition to the results of [14] is performed.

The parameters of the SR electronic spectrum can be estimated within the framework of the Kronig-Penny model [15]. Thus it is possible to determine from (11) the dispersion dependence ofω *(k*) in a wide range of temperatures, period of SR and width of potential wells forming SR.

Recently, the attention of researchers has been focused on the study of graphene SRs and, in particular, on the peculiarities of the laws of dispersion of plasma waves in such structures [19-23], which may determine the further subject of research and development of this work.

CONCLUSION

The problem of the influence of a nonlinear electromagnetic wave on the law of dispersion of plasma waves in a two-dimensional semiconductor quantum superlattice has been solved. An expression allowing to determine the dispersion dependenceω (k) in a wide range of temperatures, superlattice period and width of potential pits forming the superlattice is obtained. It is shown that the plasma frequency decreases as the amplitude of the nonlinear wave increases. In the limiting case of weak nonlinearity, an analytical expression forω (k) is obtained. Calculations are performed on the basis of the quantum theory of plasma waves in the approximation of random phases taking into account the overshooting processes.

This study has a fundamental character, the obtained theoretical results can be useful in the experimental study of SR and effectively complement the available data on collective effects in low-dimensional semiconductor superstructures.

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FIGURE CAPTIONS

Fig. 1. Geometry of the problem.

Fig. 2. Plasma wave dispersion law ω *(k)* at: *k=0*.4; T=100 K (a), *k=0*; T=100 K (b), *k=0*.4 (c); T=300 K, *k=0*; T=300 K (d).

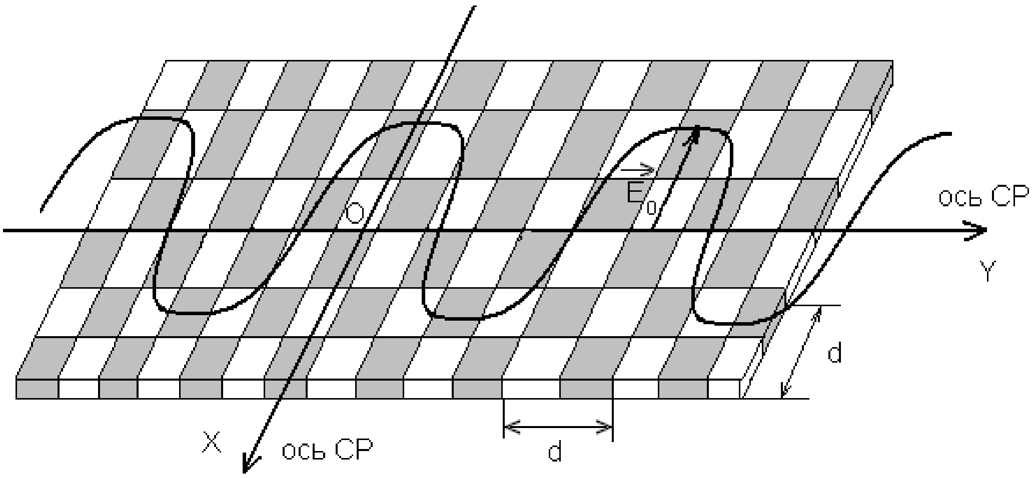

Fig. 1.

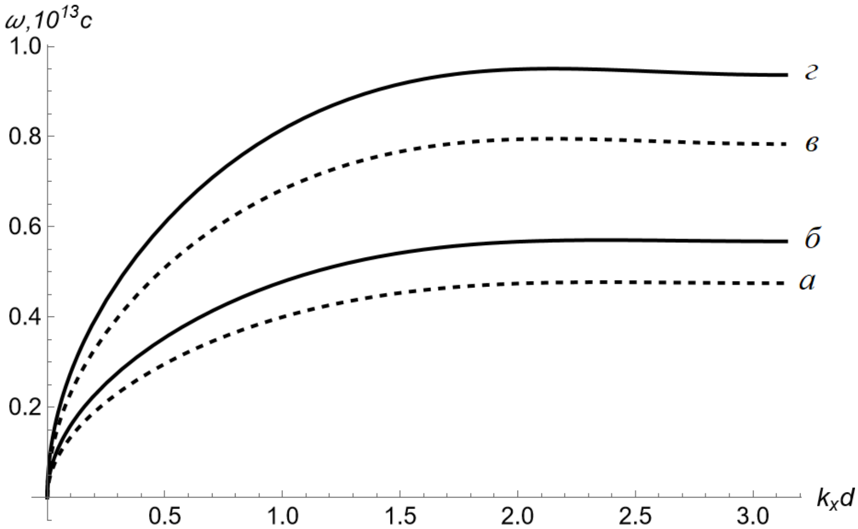

Fig. 2.