ON THE SUPERLUMINAL OBJECTS IN NON-EQUILIBRIUM MEDIA

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Abstract. An analysis of superluminal propagation of resonant and quasi-resonant soliton-like laser pulses in non-equilibrium media is presented. It is shown that such pulses are unstable. However, at the initial stage of instability development, it is possible to observe superluminal propagation of these pulse profiles due to the reshaping mechanism.

Keywords: *non-equilibrium medium, inversion population, superluminal propagation, reshaping mechanism*

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INTRODUCTION

Due to the continuous improvement of laser technology, it has become possible to generate very short optical pulses [1-4]. The duration of such pulses is of the order of several femtoseconds, which is much shorter than the typical relaxation times of the populations of quantum levels of various media. Under such conditions there appear unique opportunities to study nonequilibrium states of matter corresponding to inverse populations of quantum states. In such media, superluminal modes of propagation of laser pulses are present with necessity. Many works on this subject are known since the 1960s [5-8]. In the experimental work [5], the recorded group velocity exceeded the speed of light in vacuum by a factor of 6 - 9. In methodical reviews [6-8] the mechanisms of superluminal propagation are investigated in detail.

The present work is devoted to the analysis of superluminal propagation of pulses in resonant and quasi-resonant two-level media with inverse population of quantum states.

RESONANT SUPERLUMINAL PULSES

Propagation along the axis z of a quasi-monochromatic laser pulse in a two-level medium is described by a self-consistent system of Maxwell-Bloch (MB) equations

$$\frac{\partial r}{\partial t} = i\Delta r + i\psi w, \qquad (1)$$

$$\frac{\partial w}{\partial t} = \frac{i}{2}(\psi^* r - \psi r^*) , \qquad (2)$$

$$\frac{\partial \psi}{\partial z} + \frac{1}{c}\frac{\partial \psi}{\partial t} = -i\alpha r - i\frac{c}{2\omega}\nabla_\perp^2\psi. \qquad (3)$$

Here $\psi = d\varepsilon/\hbar$ - complex Rabi frequency, ε and r - complex envelopes of the electric field of the pulse and atomic dipole moment, respectively, w - population difference (inversion) of quantum states, d - real matrix element of the dipole moment of the quantum transition under consideration, $\hbar$ - Planck's constant, c - speed of light in vacuum, $\Delta = \omega_0 - \omega$ - detuning of the carrier frequency ω of the pulse from the resonance frequency ω_0 of the quantum transition, , $\alpha = 4\pi d^2 n\omega/\hbar c$ n - concentration of two-level atoms, $\nabla_\perp^2$ - transverse Laplacian.

In system (1) - (3), we neglected dissipative processes because we consider that the duration of τ_p pulse and the observation time of the propagation process are much shorter than all relaxation times.

The system of material equations (1), (2) has a well-known integral of motion [9]

$$w^2 + |r|^2 = w_{in}^2, \qquad (4)$$

where w_{in} is the initial difference of populations of quantum states.

Considering (4), it is easy to see that in the case of exact resonance ($\Delta = 0$) the system (1), (2) has the following solutions

$$r = iw_{in} \sin\theta , \qquad w = w_{in} \cos\theta, \qquad (5)$$

where

$$\theta = \int_{-\infty}^{t} \psi dt' , \qquad (6)$$

and ψ is a real dynamic variable.

Substituting (5) into (3) taking into account (6), we obtain in the one-dimensional case ($\nabla_{\perp}^2 \psi = 0$) the sine-Gordon (SG) equation

$$\frac{\partial^2 \theta}{\partial z \partial t} + \frac{1}{c}\frac{\partial^2 \theta}{\partial t^2} = \alpha w_{in} \sin\theta, \qquad (7)$$

which has a "kinky" solution of the form [10]

$$\theta = \sigma \arctan e^{(t - z/v)/\tau_p}, \qquad (8)$$

where

$$\sigma = 4, \qquad \frac{1}{v} = \frac{1}{c} - \alpha w_{in} \tau_p^2, \qquad (9)$$

and the time duration τ_p acts as a free parameter.

From (5), (6) and (8) we find

$$w = w_{in}\left[1 - 2\text{sech}^2\left(\frac{t-z/v}{\tau_p}\right)\right],$$

(10)

$$\psi = \psi_m \text{sech}\left(\frac{t-z/v}{\tau_p}\right), \quad \psi_m = \frac{2}{\tau_p}. \qquad (11)$$

It can be seen from (11) that the envelope of the laser pulse has the form of a soliton, the time duration of which is equal to τ_p . As follows from (10), after passing the soliton (at $t \to +\infty$), the population difference w returns to its initial value w_{in} , which it was equal to at . $t \to -\infty$ The velocity v of the soliton is determined by the second expression (9). At equilibrium initial population of quantum states $w_{in} < 0$. Then, as can be seen from (9), we have a pre-light propagation regime at which $v < c$. This mode corresponds to the self-induced transparency (SIT) effect [9, 10]. With the leading edge, the laser pulse induces the atoms from the ground state to the excited state, and with the trailing edge it also induces their return to the initial state. This causes the soliton propagation to be slower than the speed of light.

In the case of nonequilibrium initial population ($w_{in} > 0$), as follows from the second expression (9), the solution under consideration formally describes the superluminal mode of soliton propagation: $v > c$. In this case, the optical pulse at the leading edge induces the atoms from the excited state to the ground state, and the trailing edge returns the atoms to the initial excited state (see (10)). At first sight, here the speed should also be less than the speed of light, since time is spent for pumping energy from atoms to momentum and for its return back to atoms. But this reasoning is clearly inconsistent with the second formula (9), from which it follows that in this case the velocity of the soliton exceeds the velocity of light in the void. The point is that the induced emission at nonequilibrium population of quantum levels is caused by an almost imperceptible "tail" part, which is far ahead of the central peak. As a result, it generates a new peak of impulse, transferring the medium to the equilibrium state by the moment of arrival of the old peak into it. At the same time, the old peak is absorbed and the impression of superluminal propagation of the pulse maximum is created. Thus, it is not the energy of the pulse but its shape that propagates with superluminal velocity [6]. This propagation mechanism is called reshaping [11]. Obviously, in this mechanism the pulse does not carry information either.

In the experimental work [5] the registered group velocity of the light pulse exceeded the velocity of light in vacuum by a factor of 6-9. A detailed theoretical analysis of the reformation mechanism with interpretation of the results of [5] is contained in [6].

According to the McCall-Hahn area theorem [9, 12], in an equilibrium medium the pulses whose total area$A = \theta_{|t\to+\infty} = \int_{-\infty}^{+\infty} \psi dt$ is a multiple of2π are stable. In media with non-equilibrium initial population, for which$w_{in} > 0$, the pulses for which$A = \pi, 3\pi, 5\pi\ldots$ [12] are stable. It follows from (8) and the first expression (9) that in our case$A = \pi\sigma/2 = 2\pi$. Thus, in an equilibrium environment ( )$w_{in} < 0$ 2π -soliton CIP (11), for which$v < c$, is stable. In a nonequilibrium environment ($w_{in} > 0$) the superluminal considered2π -soliton is unstable. Two questions immediately arise here. 1) With what speed the stableπ -impulse propagates in a nonequilibrium medium? 2) Why, nevertheless, unstable superluminal2π -soliton was observed in the experiment?

π - IMPULSE

Following [12], we introduce the automodel variable

$$\xi = z(t - z/c). \tag{12}$$

Then (7) will take the form of an ordinary differential equation

$$\xi\theta'' + \theta' = \alpha w_{in} \sin\theta, \tag{13}$$

where "dash" denotes the derivative on the variable . ξ

Numerical analysis shows that equation (13) has a solution in which the envelope ψ has a prominent main maximum at the point$\xi = 0$ [12]. On the sides of the main maximum there are small-amplitude oscillations. The area of such an impulse is equal toπ , which corresponds to the condition of its stability in a nonequilibrium medium.

At the point of major high$\partial\psi/\partial t \sim \theta'' = 0$. Moreover, here$\xi = 0$. Therefore, given the goal of finding a solution in the neighborhood of the main maximum, we can neglect the first summand in the right-hand side of equation (13). Then we have an approximate solution of the form (8), where

$$\sigma = 2, \qquad v = c, \tag{14}$$

$$\frac{1}{\tau_p} = \alpha w_{in} z. \tag{15}$$

Thus, the area of the momentum is determined by the area of its main maximum in the neighborhood of$\xi = 0$ and is equal toπ . Using also (5), we find

$$w = -w_{in} \tanh\left(\frac{t - z/c}{\tau_p}\right). \tag{16}$$

If at the beginning (at$t = -\infty$) all atoms are excited, then$w_{in} = 1/2$. Then, as can be seen from (16), after the impulse (at$t = +\infty$) we have$w = -1/2$. I.e., all atoms transition to the ground state. Thus, as propagates, the automodel π - pulse induces atoms from the excited state to the ground state.

For the electric field envelope from (6) and (8), taking into account (14), we have the first expression (11), where

$$\psi_m = \frac{1}{\tau_p}, \tag{17}$$

and the speed and duration are determined by formulas (14) and (15), respectively.

Thus, the automodel π -impulse in a nonequilibrium medium propagates with a speed equal to the speed of light in vacuum. This is the answer to the first question posed at the end of the previous section.

As the π -pulse propagates, it experiences amplification accompanied by its self-compression. In this case, the temporal duration of the pulse decreases inversely proportional to the distance traveled, and the amplitude grows proportional to the distance.

The distance l_{inst} , at which the instability of the superluminal 2π -pulse considered in the previous section develops, can be estimated from the area theorem [9] by the formula $l_{inst} \sim \pi/\alpha T_2^*$, where T_2^* is the time characterizing the inhomogeneous broadening of the quantum transition. With the parameters of crystalline ruby used in [5], we have $l_{inst} \approx 30$ cm. At the same time, the experimental work [5] used ruby samples with sizes from 7 to 24 см . Thus, the discussed instability did not have time to develop. Therefore, a superluminal 2π -pulse propagating in the reshaping regime was observed. This is the answer to the second question posed at the end of the previous section.

The lifetime of the medium in the nonequilibrium state is ~ $10^{(-8)\,s}$. During this time, a superluminal pulse travels a distance of ~ 1 -10 м , which significantly exceeds the size of the samples used in [5]. Therefore, the spontaneous relaxation of the nonequilibrium medium to the equilibrium state could be neglected with good accuracy.

QUASI-RESONANT SUPERLUMINAL PULSES

Let us now consider optical pulses propagating under conditions of quasi-resonance [13-15]

$$\delta = (\Delta\tau_p)^{-1} << 1. \qquad (18)$$

It is clear that at such a large detuning of Δ from the resonance the excitation of atoms is weak, i.e., w should be insignificantly different from w_{in} . Carrying out in (1) the expansion on the small parameter δ [13 - 15], we have

$$r = -\frac{\psi}{\Delta} w + i\frac{w_{in}}{\Delta^2}\frac{\partial\psi}{\partial t} + \frac{w_{in}}{\Delta^3}\frac{\partial^2\psi}{\partial t^2}\,. \qquad (19)$$

From (4) with accuracy up to terms $\sim|r|^2$ we find

$$w = w_{in}\left(1 - \frac{|r|^2}{2w_{in}^2}\right).$$

Substituting here instead of r the first summand from expansion (19) using substitution $w \to w_{in}$, we obtain

$$w = w_{in}\left(1 - \frac{|\psi|^2}{2\Delta^2}\right). \qquad (20)$$

From (19) and (20) we come to the expression

$$r = -w_{in}\frac{\psi}{\Delta}\left(1 - \frac{|\psi|^2}{2\Delta^2}\right) + i\frac{w_{in}}{\Delta^2}\frac{\partial\psi}{\partial t} + \frac{w_{in}}{\Delta^3}\frac{\partial^2\psi}{\partial t^2}. \qquad (21)$$

After substituting (21) into (3) and simple transformations we have

$$i\frac{\partial \Phi}{\partial z} = g|\Phi|^2\Phi + \frac{\beta}{2}\frac{\partial^2\Phi}{\partial \tau^2} + \frac{c}{2\omega}\nabla_\perp^2\Phi, \qquad (22)$$

where

$$\Phi = \psi e^{-i\alpha w_{in} z/\Delta}\ , \qquad (23)$$

$g = \alpha w_{in}/2\Delta^3$, , $\beta = 2\alpha w_{in}/\Delta^3$ $\tau = t - z/v_g$, and the linear group velocity v_g is defined by the expression

$$\frac{1}{v_g} = \frac{1}{c} - w_{in}\frac{\alpha}{\Delta^2}. \qquad (24)$$

If we neglect the third summand in the right part of (22), we obtain a one-dimensional nonlinear Schrödinger equation (NLS). The coefficients g and β in this equation have the same signs. In this case, the NUSH has stable solutions in the form of "light" solitons propagating with linear group velocity v_g [16]. From (24) we can see that in a nonequilibrium ($w_{in} > 0$) environment $v_g > c$. In this case, as in the case of exact resonance, the superluminal propagation occurs in the reshaping regime. Therefore, there are no contradictions with the principles of the theory of relativity.

One-dimensional superluminal NUSH solitons can be observed at propagation distances smaller than the diffraction broadening length of these solitons. Therefore, it is important to consider the stability of quasi-resonant three-dimensional localized pulses - spatio-temporal solitons or light bullets [17 - 20]. For illustration, let us consider the case $g, \beta > 0$.

Following [19, 20], we perform the Madelung transformation

$$\psi_1 = \sqrt{\rho}\, exp(-i\omega\varphi/c), \qquad (25)$$

where ρ and φ are functions to be defined. Substituting (25) into (22), we come to the system of equations formally describing the flow of an imaginary quantum fluid:

$$\frac{\partial\rho}{\partial z} + \nabla(\rho\nabla\varphi) = 0, \qquad (26)$$

$$\frac{\partial\varphi}{\partial z} + \frac{(\nabla\varphi)^2}{2} - \frac{c}{\omega} g\rho = \left(\frac{c}{\omega}\right)^2 \frac{\nabla^2\sqrt{\rho}}{2\sqrt{\rho}}, \qquad (27)$$

where $\nabla^2 = \nabla_\perp^2 + \partial^2/\partial\eta^2$ is the effective three-dimensional Laplacian,

$$\eta = \sqrt{\frac{c}{\omega\beta}}\tau, \qquad (28)$$

∇ - is the effective three-dimensional gradient operator in the variables $\boldsymbol{r}_\perp$ and , η $\boldsymbol{r}_\perp$ is the radius-vector transverse to the direction of momentum propagation.

The hydrodynamic approach based on a system like (26), (27) is very effective in the theory of self-focusing and light bullet formation [17 - 23].

The continuity equation (10) has an automodel "spherically symmetric" solution in the coordinate system (, $\mathbf{r}_\perp$ η) [21]

$$\rho = \psi_m^2 \frac{R_0^3}{R^3} exp(-\zeta^2/R^2), \qquad \varphi = f(z) + \frac{\zeta^2}{2}\frac{R'}{R}, \qquad (29)$$

where , $\zeta = \sqrt{r_\perp^2 + \eta^2}$ $R = R(z)$ is the characteristic size of the light energy cluster under consideration, ψ_m is the field amplitude, R_0 is the equilibrium value of the parameter , R $f(z)$ is some function, the dash over the variable R here and below denotes the derivative on the variable . z

Following [22], we use the near-axis approximation (near-axis approximation) in the left-hand side of equation (27), i.e., we write $e^{-r^2/R^2} \approx 1 - r^2/R^2$. Equating then in the left and right parts of the expressions at r^0 and r^2 , we come to the equations

$$f' = \frac{c}{\omega} g\psi_m^2 \frac{R_0^3}{R^3} - \frac{3}{2}\left(\frac{c}{n\omega}\right)^2 \frac{1}{R^2}\,, \qquad (30)$$

$$R'' = -\frac{\partial U}{\partial R} = \left(\frac{c}{n\omega}\right)^2 \frac{1}{R^3} - \frac{2c}{\omega} g\psi_m^2 \frac{R_0^3}{R^4}. \qquad (31)$$

Equation (31) is an equation of motion of a Newtonian particle of unit mass in an external field with "potential energy" $U(R)$ where R and z play the roles of particle coordinates and time, respectively.

The first summand in the right part of (31) corresponds to diffraction effects. In turn, the second summand describes the effect of cubic (Kerr) nonlinearity.

The conditions for the formation of a stable spatiotemporal soliton are $(\partial U/\partial R)_{R=R_0} = 0$, $(\partial^2 U/\partial R^2)_{R=R_0} > 0$, which corresponds to the presence of a local minimum in the dependence $U(R)$ at the equilibrium value of the bullet radius. From (31) it is easy to see that it is impossible to satisfy these conditions, since the dependence $U(R)$ does not have a local minimum. On the contrary, this dependence has a local maximum. This conclusion is consistent with the known fact: at Kerr nonlinearity alone, three-dimensional space-time solitons are unstable [24].

CONCLUSION

The methodological consideration carried out in the present work shows that superluminal optical pulses in nonequilibrium (amplifying) media are unstable, as well as nonequilibrium media themselves. This conclusion is consistent with the conclusions of previous works, including [6] and [8], although the stability issues are investigated here in other ways. On the other hand, the reshaping mechanism, which does not contradict fundamental physical principles, allows us to observe superluminal impulses at distances smaller than the characteristic lengths of instability development.

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