DIFFERENTIAL GAIN OF THz Radiation In Crystalline Quartz Plate In The Field Of Pump Wave

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Abstract. The possibility to exploit nonlinear Fabry-Perot interferometers to differential gain of terahertz radiation in the field of a pump wave of the same frequency was theoretically considered. It is shown that in mirrorless nonlinear Fabry-Perot interferometer consisted of crystalline quartz plate, which reflection is determined by Fresnel reflection only, the regime of maximal differential gain of radiation with central frequency at 1 THz can be observed at thickness of working medium near 1 mm and at radiation intensity with order of magnitude at 10^8 W · cm^{-2}.

Keywords: *nonlinear Fabry-Perot interferometer, THz spectral range, mirrorless interferometer, crystalline quartz, optical transistor, differential gain*

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INTRODUCTION

Nonlinear transmittance of interferometers filled with various optical media in the field of intense light was experimentally demonstrated back in the 1970s [1].

Hysteresis and bistable and multistable transmission modes of nonlinear interferometers attracted special attention of researchers. These phenomena have attracted great interest not only because of their fundamental importance, but also because of the inspiring prospects of practical applications for "light by light" control (see, for example, the review of more than a thousand publications in the monograph [2]). Interferometers filled with materials characterized by high and low-inertia nonlinearity in optical performance have been shown to be of great importance for applications. However, materials whose nonlinear response time would correspond to the subpicosecond range, competitive for the creation of radiation parameter control devices in comparison with electronic analogs, and whose nonlinearity would be observed at relatively low radiation intensities, were not found at that time [3]. This prevented the realization of promising scientific projects, such as, for example, the creation of purely optical ultrafast digital computing systems [4].

The recent discovery of a number of media in the terahertz (THz) spectral range of a giant and simultaneously low-inertia nonlinearity of the refractive index of vibrational nature, millions of times higher than the nonlinearity of these media in the visible and near-infrared ranges [5-13], again draws attention to the possibility of creating a variety of systems for ultrafast control of "light with light", but now in the THz spectral range [14, 15].

An important feature of many optical media in the THz spectral range is not only the high nonlinearity of the refractive index, but also the significant magnitude of its linear part. This allows us to consider nonlinear Fabry-Perot interferometers in their simplest form of a crystalline plate, for which the Fresnel reflection quite allows us to observe significant effects of multibeam interferometry without sputtering of mirrors on the plate faces.

The present work is devoted to analyzing the possibility of using mirrorless nonlinear Fabry-Perot interferometers for ultrafast differential amplification of THz radiation in the pump wave field of the same frequency. By differential amplification in this paper we mean a large increase in the output signal intensity ΔI_{out} with a small increase in the signal intensity at the input to the nonlinear interferometer ΔI_{in}. The energy required to amplify the signal comes from a pump wave of the same frequency. Highly efficient differential amplification is assumed when $\Delta I_{out} \gg \Delta I_{in}$.

The paper shows that to observe in a nonlinear Fabry-Perot interferometer the mode of maximum nonlinear amplification of any signal, including terahertz, it is necessary that the geometric thickness of the working medium and the radiation intensity at the input to the interferometer take fixed discrete values, which are determined by the Fresnel reflection coefficient of the medium and the coefficient of its nonlinear refractive index. It has been demonstrated by calculations that in crystalline quartz, which has a giant and low-inertia refractive index nonlinearity in the THz range, the differential amplification mode for radiation with a center frequency of 1 THz can be observed at thicknesses of the order of 1 mm and intensities of the order of $10^8 W \cdot cm^{-2}$.

BISTABILITY AND GAIN IN A NONLINEAR FABRY-PEROT INTERFEROMETER

The transmittance function of a nonlinear Fabry-Perot interferometer consisting of two plane-parallel mirrors, between which the optical medium is enclosed, in the case of normal incidence of radiation on the interferometer, has the following form [2]

$$I_t = \frac{I_0}{1+\frac{4R}{(1-R)^2}\sin^2\left(\frac{2\pi L}{\lambda}n_0+\frac{3}{2}\frac{1+R}{1-R}\frac{2\pi L}{\lambda}n_2 I_t\right)} \quad (1)$$

where $I_0$and $I_t$are the radiation intensities at the input and output of the interferometer, respectively, λ is the wavelength of radiation, *L* is the geometric thickness of the optical medium of the interferometer, $n_{(0)}$is its linear refractive index, $n_2$is the coefficient of the nonlinear refractive index of the medium, *R* is the reflection coefficient of the interferometer mirrors.

Formula (1) is also valid for describing the transmittance of a mirrorless interferometer in which the reflection of incident radiation occurs not from the mirrors but from the interface between the optical medium of the interferometer and air. In such a case, the reflection coefficient *R is* calculated by Fresnel formulas and depends only on the refractive index of the medium. For the first time, such a nonlinear mirrorless interferometer was experimentally considered in 1979 for monochromatic radiation of the near infrared spectral range [16].

Fig. 1 shows in logarithmic scale the transmission curve of 1 THz radiation calculated by formula (1) by a mirrorless nonlinear Fabry-Perot interferometer in the form of a plate of crystalline quartz with thickness *L*= 1.013 mm (refractive index n_0= 2.1 and, accordingly, Fresnel reflection coefficient *R*= 0.126). At input intensities greater than a value of $1 \cdot 10^8 \mathrm{W} \cdot \mathrm{cm}^{-2}$,the nonlinear transmittance increases dramatically, exceeding what it would be with linear refraction. In this range of input intensities, it is possible to realize the differential amplification mode of a weak signal of the same frequency (see inset in the figure). Sometimes this mode is also called "transistor mode" because of the similarity of the interferometer transmission curve at differential amplification of the input radiation in the pump wave field to the output characteristic of a bipolar transistor. At intensities of the order of $3 \cdot 10^8 W \cdot cm^{-2}$and higher in the transmission function of the interferometer, transmission bistability is observed, when one value of intensity at the input to the interferometer corresponds to two values at its output (strictly speaking, one value of input intensity on the transmission curve corresponds to three values, but one of them - the middle one - is unstable and is not realized in practice [2]). It is important to note that bistable transmission modes occur after the "transistorized" mode. In this sense, the point where the maximum gain is observed is a threshold point for the observation of bistability as well. However, for example, at a slightly larger thickness of the crystalline quartz plate *L*= 1.063 mm, there is no differential gain mode in the transmission curve of the nonlinear interferometer, and only the bistability mode is observed. Mathematically, the conditions for the presence of the differential gain mode in the transmission function of a nonlinear Fabry-Perot interferometer are given by the expressions [2]

$$\frac{dI_0}{dI_t} = 0, \ \frac{d^2 I_t}{d{I_0}^2} = 0 \qquad (2)$$

For further analysis and calculations, it is convenient to introduce normalized variables:

$$\begin{cases} \tilde{L} = \frac{2\pi L}{\lambda} n_0 \\ \tilde{I}_t = \frac{3}{2}\frac{1+R}{1-R}\frac{n_2}{n_0}\tilde{L} I_t \\ \tilde{I}_0 = \frac{3}{2}\frac{1+R}{1-R}\frac{n_2}{n_0}\tilde{L} I_0 \end{cases} \qquad (3)$$

In dimensionless variables (3) the transmittance function (1) takes a simpler form

$$\tilde{I}_0 = \tilde{I}_t \left[1 + \frac{4R}{(1-R)^2}\left(\tilde{I}_t + \tilde{L}\right)^2\right] \tag{4}$$

Substituting equation (4) into expressions (2) in the approximation of no intensity dependence of the reflection coefficient leads to a system of two linearly independent equations

$$\begin{cases} \frac{1+R^2}{2R} - \cos 2\left(\tilde{I}_t + \tilde{L}\right) + 2\tilde{I}_t \sin 2\left(\tilde{I}_t + \tilde{L}\right) = 0 \\ \sin 2\left(\tilde{I}_t + \tilde{L}\right) + \tilde{I}_t \cos 2\left(\tilde{I}_t + \tilde{L}\right) = 0 \end{cases} \tag{5}$$

System (5) reduces to Eq.

$$\cos 2\left(\tilde{I}_t + \tilde{L}\right) = f_{\pm}(R) \tag{6}$$

where

$$f_{\pm}(R) = -\frac{R^2+1 \pm \sqrt{R^4+34R^2+1}}{4R} \tag{7}$$

Since the cosine in (6) cannot modulo exceed one, only the root of $f_{-}(R)$ satisfies the solution of equation (6). Further, for convenience, let us re-define $f_{-}(R) \equiv f(R)$. Then the solution of the system (5) with respect to the unknowns $\tilde{I}_t$ and $\tilde{L}$ is represented in the form

$$\begin{cases} \tilde{I}_t = f(R)^{-1}\sqrt{1-f(R)^2} \\ \tilde{L} = -f(R)^{-1}\sqrt{1-f(R)^2} - \frac{1}{2}\arccos f(R) + \pi m \end{cases} \tag{8}$$

where m is an integer.

From expressions (8) for the values normalized output radiation intensity $\tilde{I}_t$ and thickness of the interferometer working medium $\tilde{L}$, at which the observation of differential amplification is possible, we obtain the ratios of non-normalized values

$$\begin{cases} I_0 = \frac{\lambda}{2\pi}\frac{1}{Ln_2}\Phi(R) \\ L = \frac{\lambda}{2\pi n_0}(\Psi(R) + \pi m) \end{cases} \quad (9)$$

where

$$\Phi(R) = \frac{2}{3}\frac{1-R}{1+R}\frac{\sqrt{1-(f(R))^2}}{f(R)}\left(1 + \frac{2R(1-f(R))}{(1-R)^2}\right) \quad (10)$$

$$\Psi(R) = -\frac{\sqrt{1-(f(R))^2}}{f(R)} - \frac{1}{2}\arccos f(R) \quad (10)$$

It is important to note that the expression$\Phi(R)$ in (9) has the form of a decay integral, which gives an estimate of the conditions for the onset of small-scale self-focusing of radiation in a nonlinear medium [17] and, accordingly, the limits of applicability of the plane transversely homogeneous wave approximation used in this paper. The estimates are known [18]: the effect of spatial instability of a plane transversely homogeneous wave begins to appear in a nonlinear medium when the value of the decay integral is greater than π.

QUARTZ CRYSTAL PLATE IN THE FIELD OF TGZ RADIATION

In experimental work [12] it was shown that crystalline quartz possesses in the THz range a nonlinear refractive index coefficient n_2= 5· 10^{-10}cm^2· W^{-1}, which is about one million times greater than its value for this material in the optical range [19]. In addition to the giant and low-inertia nonlinearity of vibrational nature, crystalline quartz has in the THz range good transparency and relatively large linear refractive index n_0= 2.1 (at a wavelength of 1 THz), which gives the value of the reflection coefficient of Fresnel "mirrors" R= 0.126. It is important that at such a relatively small$R$ decay integral$\Phi(R)$ has a value less than π, i.e., a plate of crystalline quartz can be considered as a mirrorless Fabry-Perot interferometer in the field of a flat monochromatic THz wave.

Fig. 2 shows the results of calculation by formulas (8) of the thickness of the crystalline quartz plate and the intensity of THz radiation at the entrance to this plate, necessary to observe the maximum differential amplification of a weak signal in the field of an intense pump wave at a frequency of 1 THz. The figure shows that the allowed thicknesses and intensities take values from a discrete set. The discreteness of the values is due to the integer parameter *m* included in the expressions (9). Each value of *m* corresponds to a different thickness of the optical medium and input intensity. In this case, the greater the thickness of the quartz plate, the lower the input intensity is required to observe the amplification effect.

At m= 15, the required quartz plate thickness is L (m= 15)= 1.013 mm, and the input intensity $I_0(m= 15)= 1.1\cdot 10^8 \mathrm{W}\cdot \mathrm{cm}^{-2}$. It is at these values that we observed the maximum differential gain in Fig. 1.

Let us estimate the differential gain of what order is possible to achieve in a nonlinear mirrorless Fabry-Perot interferometer based on crystalline quartz. For this purpose, let us imagine that a rectangular signal is incident on the nonlinear interferometer in a manner similar to that depicted in Fig. 1. Suppose that the minimum non-zero intensity of such a pulse is$I_{min} = 0.9 I_0(m)$, and the maximum intensities are$I_{max} = 1.1 I_0(m)$ and$\Delta I_0 = I_{max} - I_{min} = 0.2 I_0(m)$. As a result of the pulse passing through the interferometer, the maximum and minimum values of its output intensity will change, and their difference will beΔI_t . Fig. 3 visualizes the considered ranges for the case$I_0 = I_0(m = 15)$. In that case$\Delta I_0 = 2.236 \cdot 10^7$ Вт $\cdot$ см$^{-2}$

and$\Delta I_t = 5.890 \cdot 10^7$ Вт $\cdot$ см$^{-2}$. As the thickness ofI_0 , according to Fig. 2, bothΔI_0 , andΔI_t decrease, and vice versa. For example, for $m = 20 \Delta I_t = 4.355 \cdot 10^7$ Вт $\cdot$ см$^{-2}$, and for $m = 5 \Delta I_t = 1.825 \cdot 10^8$ Вт $\cdot$ см$^{-2}$. The ratio of$\frac{\Delta I_t}{\Delta I_0}$ is generally inconstant. For small$m$, when$\frac{dI_0(m)}{dm}$ is large,$\frac{\Delta I_t}{\Delta I_0}$ changes a lot. When$m$ is large, the derivative of$\frac{dI_0(m)}{dm}$ is small, so$\frac{\Delta I_t}{\Delta I_0}$ is almost constant. For example$\frac{\Delta I_t(m=15)}{\Delta I_0(m=15)} \approx \frac{\Delta I_t(m=20)}{\Delta I_0(m=20)} \approx 2.63$, while .$\frac{\Delta I_t(m=5)}{\Delta I_0(m=5)} \approx 2.4$

An important aspect in the experimental use of signal amplification in a nonlinear Fabry-Perot interferometer is the sensitivity of the maximum differential gain mode to changes in the interferometer parameters. Figure 4 illustrates the sensitivity of the interferometers to changes in the thickness of the optical medium. It can be seen that when the thickness is changed by 1 μm in a mirrorless interferometer with Fresnel reflectance *R*= 0.126, the point of maximum differential gain shifts by 2% in the input intensity. At the same time, in an interferometer with mirrors with reflection coefficient *R*= 0.99, when the thickness is changed by 0.1 μm, the interferometer transmittance changes radically and observation of the differential gain mode becomes impossible. Thus, mirrorless interferometers have an advantage over interferometers with mirrors in terms of the required accuracy of ensuring the thickness of the optical medium to observe the effect of maximum differential gain.

CONCLUSION

Thus, we have demonstrated the possibility of using a plate of crystalline quartz as a mirrorless nonlinear Fabry-Perot interferometer in the field of THz radiation. The conditions for observing differential amplification of THz radiation in the field of a pump wave of the same frequency by such an interferometer have been determined. It is shown that to realize the amplification, the thickness of the optical medium of the interferometer and the amount of radiation at the entrance to the interferometer must take specific discrete values determined by the value of the plate reflection coefficient and the coefficient of its nonlinear refractive index.

It is shown that in crystalline quartz, which has a giant and low-inertia refractive index nonlinearity in the THz range, the maximum nonlinear amplification mode for radiation with a center frequency of 1 THz can be observed at thicknesses of the order of 1 mm and intensities of the order of $10^8 W\cdot cm^{-2}$. Estimates of the magnitude of differential amplification of radiation by the interferometer according to the change in the difference between the maximum and minimum values of the intensity ΔI_t of the pulse at the exit of the interferometer with respect to the change in the difference between the maximum and minimum values of the intensity ΔI_0 of the pulse at the entrance to the interferometer showed that for crystalline quartz the ratio ΔI_t to ΔI_0 for all thicknesses satisfying the condition of maximum amplification varies in the range of 2.4 - 2.6 units.

It is revealed that mirrorless interferometers are less sensitive to changes in the thickness of the optical medium than interferometers with mirrors. This makes mirrorless interferometers promising for creating optical devices based on them.

The results obtained indicate that the nonlinear Fabry-Perot interferometer can be used to control THz band radiation due to differential gain, including as an optical transistor when using high-intensity radiation and a weak control pulse. The evaluations presented in this paper may be useful in the design of optical transistors in the THz spectral range.

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FIGURE CAPTIONS

Fig. 1. Dependence of the intensity of radiation with a frequency of 1 THz at the output of a 1.013 mm thick quartz crystal plate on the intensity of radiation at the input to the plate (solid orange curve). The inset illustrates the possibility of using a quartz crystal plate to amplify an input signal of the same frequency (A - time profile of the input signal, B - of the output signal). The blue dashed line shows a view of the linear transmittance.

Fig. 2. Values of the input radiation intensityI_0 at the center frequency of 1 THz and the corresponding thickness of the quartz crystal plate$L$, at which the maximum

differential amplification of signals is observed. Integer numbers denote the ordinal number of the plate thickness values satisfying the condition of maximum amplification.

Fig. 3. Model of signal amplification in a nonlinear interferometer based on a plate of crystalline quartz. ΔI_0 - is the amplitude of the input signal,ΔI_t is the amplitude of the output signal.

Fig. 4. Transmission function of a nonlinear Fabry-Perot interferometer with a working medium made of crystalline quartz: without mirrors (*R* = 0.126) (a) and with mirrors (*R* = 0.99) (b). The solid curves correspond to the thicknesses of the optical medium necessary to observe the effect of maximum differential gain. Dashed and dashed curves correspond to small deviations in thickness from the values at which the maximum differential gain is observed.

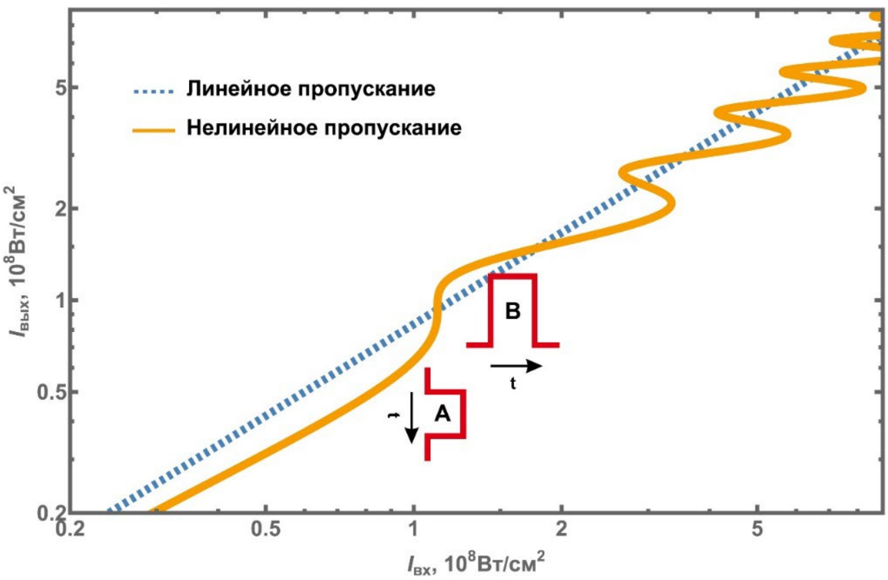

Fig. 1.

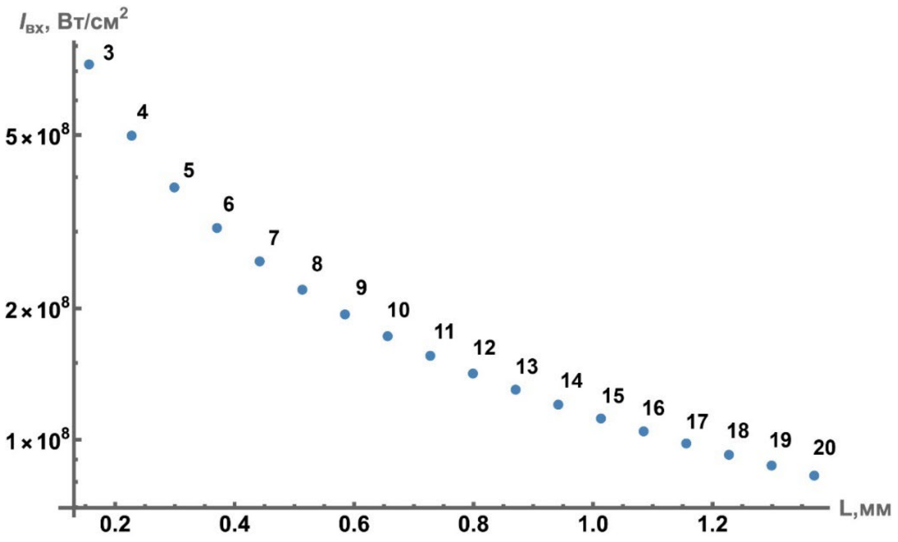
$I_{вх}$, Вт/см^2
5×10^8
2×10^8
1×10^8
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
0.2
0.4
0.6
0.8
1.0
1.2
L,мм

Fig. 2.

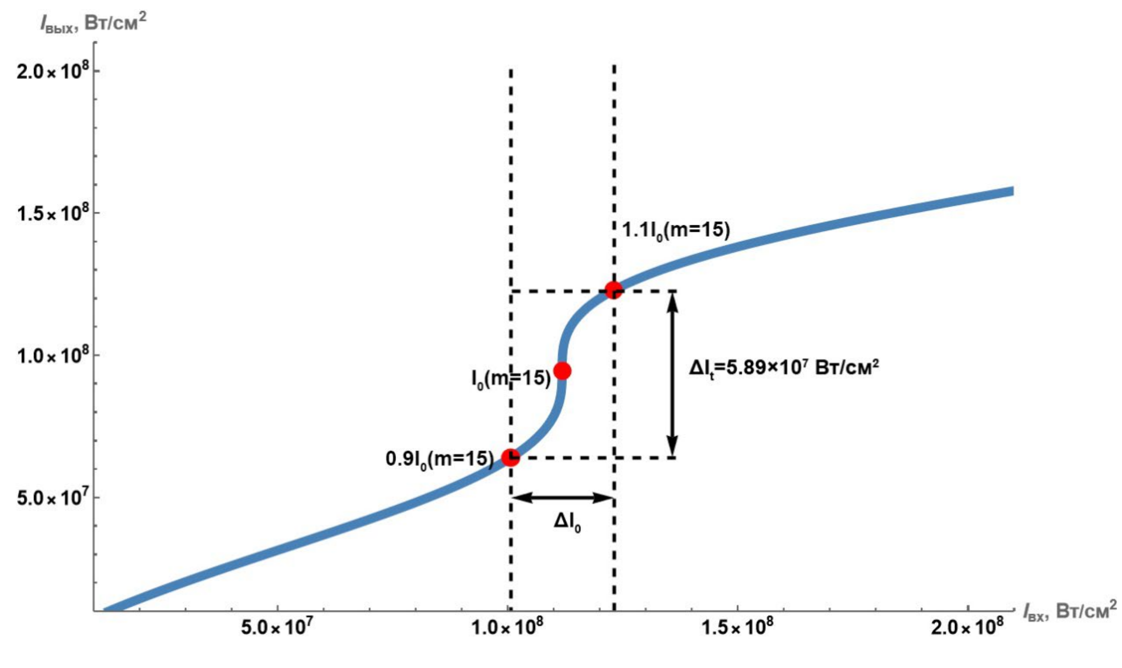
$I_{вых}$, Вт/см^2
2.0×10^8
1.5×10^8
1.0×10^8
5.0×10^7
1.1I$_0$(m=15)
I$_0$(m=15)
0.9I$_0$(m=15)
ΔI$_t$=5.89×10^7 Вт/см^2
ΔI$_0$
5.0×10^7
1.0×10^8
1.5×10^8
2.0×10^8
$I_{вх}$, Вт/см^2

Fig. 3.

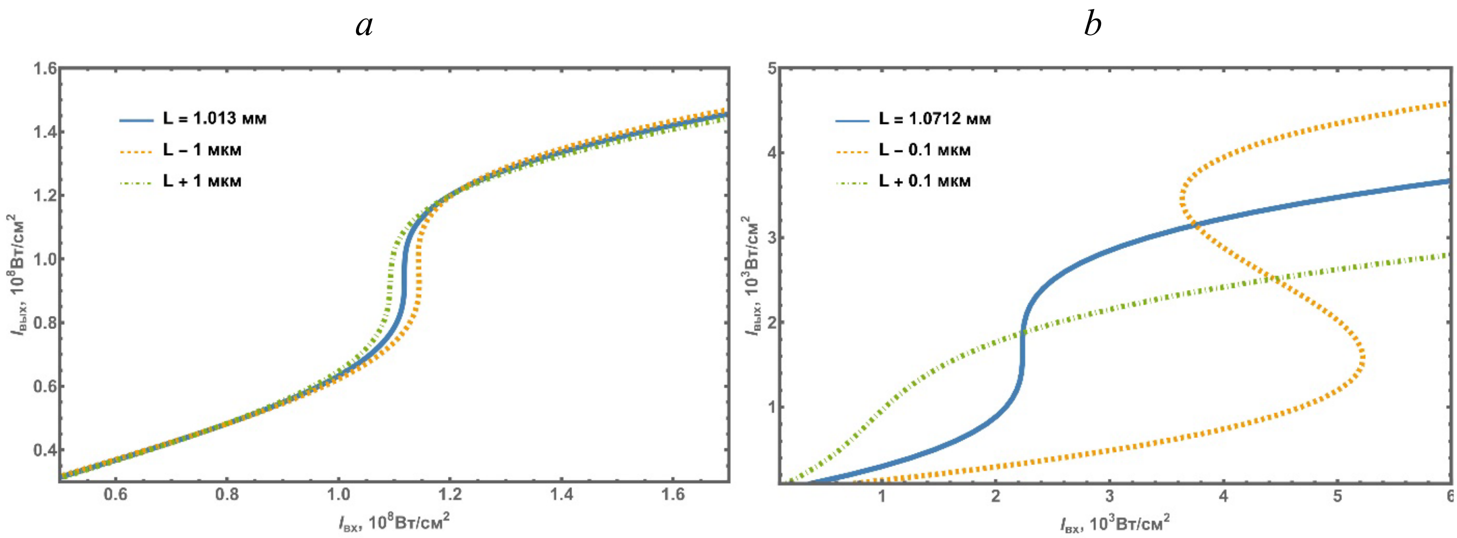
a
L = 1.013 мм
L – 1 мкм
L + 1 мкм
I вых, 10^8 Вт/см^2
I вх, 10^8 Вт/см^2
b
L = 1.0712 мм
L – 0.1 мкм
L + 0.1 мкм
I вых, 10^3 Вт/см^2
I вх, 10^3 Вт/см^2

Fig. 4.