MEAN FIELD OF ACOUSTIC WAVES WITH DISCONTINUITIES IN RANDOMLY INHOMOGENEOUS MEDIA

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Abstract. The features of the construction of closed equations for the mean field of acoustic waves with discontinuous profiles in a randomly inhomogeneous medium are considered. Different approaches to obtaining such equations are compared. It is shown that, despite the smoothing of profiles in the average, the presence of a discontinuity in the profile should be considered before the averaging operation. An exact expression for the mean field of the initial N-wave is obtained.

Keywords: *discontinuous waves, random inhomogeneous media, mean field, averaging, exact solution*

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INTRODUCTION

The problem of acoustic wave propagation in randomly inhomogeneous media and the need to calculate the statistical characteristics of such waves arise in many cases [1, 2], in particular, when sound propagates in a turbulent atmosphere [3-7]. In this case, the medium parameters change with time, so that a small set of realizations does not give a complete picture of the possible character of the wave evolution. A full-fledged description is possible only on the basis of statistical characteristics such as probability distributions or at least mean values, dispersions, etc. Let us also note the tasks of probing and restoring the parameters of inhomogeneous media, in the realization of which the noise and fluctuations present may be useful [8, 9]. At present, the issues of propagation of nonlinear waves and beams [10] in randomly inhomogeneous media, including acoustic shock waves with a narrow front from promising civil supersonic airplanes [11, 12], as well as in medical applications [13], are becoming topical. Therefore, it is necessary to develop methods for calculating the statistical characteristics of shock and burst waves in a randomly inhomogeneous medium.

The construction of exact dynamic solutions for nonlinear equations presents great difficulties, all the more so for stochastic equations with random functions. One of the effective approaches to calculation of statistical characteristics of wave fields is the averaging of stochastic equations in order to obtain equations for moments - mean field, dispersion, etc.

This paper is devoted to further refinement of the averaging procedure for acoustic shock and burst waves with narrow shock fronts. As is known [14-16], averaging in general leads to the appearance of so-called turbulent attenuation and, consequently, smoothing of shock fronts. Consequently, one would expect that the averaging of discontinuous profiles would have no peculiarities compared to the averaging of smooth profiles. However, it turns out that this is not the case, and the presence of a discontinuity must be taken into account before the averaging procedure.

MEAN-FIELD METHOD

One of the common methods of obtaining closed-form equations for averaged characteristics is the mean-field method, which has a long history [17]. It has proved itself quite well when solving linear problems. When considering nonlinear problems, the problem of closure of nonlinear summands arises. According to the mean-field method, the mean value of the square of the acoustic field (e.g., pressure), is replaced by the product of mean values. In fact, this means neglecting the mean square of pressure fluctuations, which leads to certain errors [14, 18]. In addition, it is necessary to determine how correctly it accounts for shock fronts and discontinuities in the wave profile.

As an initial equation, consider a simple wave type equation including a random disorder of the sound velocity $\varsigma(z)=\frac{c_0^2}{2}\left(c^{-2}(z)-c_0^{-2}\right)$, caused by fluctuations of the propagation medium parameters:

$$\frac{\partial p}{\partial z}-\frac{\varsigma(z)}{c_0}\frac{\partial p}{\partial \tau}-\frac{\varepsilon}{\rho c_0^3}p\frac{\partial p}{\partial \tau}=0\,, \tag{1}$$

where p *is* the acoustic pressure, z *is* a coordinate, $\tau=t-z/c_0$ is time in the accompanying coordinate system, $c(z)$ is the random local sound velocity, c_0 is the characteristic average sound velocity, ε is a nonlinear parameter, ρ is the density of the medium.

Applying the mean-field method to equation (1), we obtain the Burgers equation for the mean pressure:

$$\frac{\partial\langle p\rangle}{\partial z}-\frac{\varepsilon}{\rho c_0^3}\langle p\rangle\frac{\partial\langle p\rangle}{\partial\tau}=\frac{\sigma^2}{2c_0^2}\frac{\partial^2\langle p\rangle}{\partial\tau^2}. \qquad (2)$$

Here, the angle brackets denote ensemble averaging, σ^2 makes sense of the dispersion of the velocity tuning fluctuations (the correlation function $\langle\varsigma(z_1)\varsigma(z_2)\rangle=\sigma^2\delta(z_2-z_1)$) is specifically defined. As can be seen, the averaging has led to the appearance of so-called turbulent damping, i.e., the field is damped on average. Equation (2) is remarkable in that by replacing Hopf-Cole $V=2\Gamma\frac{\partial}{\partial\theta}\ln U$ it is reduced to a linear equation:

$$\frac{\partial U}{\partial x}=\Gamma\frac{\partial^2 U}{\partial\theta^2}.$$

Dimensionless variables are introduced here

$$V=\frac{\langle p\rangle}{p_0},\,,\,,\,,\,,\ \theta=\frac{\tau}{\tau_0}\ \ x=\frac{z}{z_{nl}}\ \ z_{nl}=\frac{\rho c_0^3\tau_0}{\varepsilon p_0}\ \ \Gamma=\frac{\sigma^2}{2c_0^2}\frac{z_{nl}}{\tau_0^2} \qquad (3)$$

where p_0 and τ_0 are the characteristic amplitude and duration of the pulse.

We will consider the N-wave as the initial signal,

$$F(\tau)=\begin{cases}-p_0\tau/\tau_0, |\tau|<\tau_0,\\ 0, |\tau|<\tau_0,\end{cases} \qquad (4)$$

representing a model version of characteristic profiles registered from supersonic aircraft [3, 5]. For the initial profile (4) we obtain the solution of the Burgers equation in dimensionless variables:

$$\frac{\langle p\rangle}{p_0}=2\Gamma\frac{\partial}{\partial\theta}\ln\left\{1+\frac{1}{2}\Phi\left(\frac{\theta-1}{2\sqrt{\Gamma x}}\right)-\frac{1}{2}\Phi\left(\frac{\theta+1}{2\sqrt{\Gamma x}}\right)+\right.$$
$$\left.+\frac{1}{2\sqrt{x+1}}\exp\left(\frac{x+1-\theta^2}{4\Gamma(x+1)}\right)\left[\Phi\left(\frac{x+1+\theta}{2\sqrt{\Gamma x}\sqrt{x+1}}\right)-\Phi\left(\frac{\theta-x-1}{2\sqrt{\Gamma x}\sqrt{x+1}}\right)\right]\right\}, \qquad (5)$$

where $\Phi(t)=\frac{2}{\sqrt{\pi}}\int_0^t e^{-t^2}dt$ is the integral of errors.

From formula (5), one can see the dynamics of the obtained front - there is a blurring of shock fronts proportional to both the dispersion of phase fluctuations and the distance traveled. In particular, even discontinuous profiles within the framework of this model are smoothed and do not contain features, which is the basis for the assumption of applicability of standard approaches to averaging of waves with discontinuities. Characteristic profiles of the solution (5) are shown in Fig. 1 for the value $\Gamma = 0.05$. There is a blurring of the wave front, both due to diffusive blurring of the initial front width and to a certain RMS drift of the mean position of the shock front. Although the solution (5) contains significant information about nonlinearity of the medium, it still does not satisfactorily describe the mean field because it is based on an approximate averaging model [14, 18].

AVERAGING OF THE EXACT DYNAMIC SOLUTION

To evaluate the accuracy of the methods described above, let us return to equation (1). It is convenient for analysis because we can construct its exact analytical solution even in the presence of fluctuations. Averaging of this solution will show the accuracy and closeness to the correct result of the solutions obtained by approximate methods.

Let's replace the variables

$$\tau_1 = \tau + \frac{1}{c_0}\int_0^z \varsigma(z')dz'$$

and reduce equation (1) to the standard simple wave equation:

$$\frac{\partial p}{\partial z} - \frac{\varepsilon}{\rho c_0^3} p \frac{\partial p}{\partial \tau_1} = 0 . \qquad (6)$$

The solution of equation (6) with an arbitrary initial profile $p(z=0) = F(\tau_1)$ is given in implicit form:

$$p = F\left(\tau_1 + \frac{\varepsilon}{\rho c_0^3} pz \right) . \qquad (7)$$

Introducing the notation $\eta = \frac{1}{c_0} \int_0^z \varsigma(z') dz'$, we write the solution of equation (1) containing fluctuations in the following form:

$$p = F\left(\tau + \eta + \frac{\varepsilon}{\rho c_0^3} pz \right) . \qquad (8)$$

The solution (8) is given in implicit form and does not allow us to average it directly. Therefore, we go to the spectrum of the wave and average it, and then find profile of the averaged wave. It is known that before the formation of a discontinuity, the spectrum of a simple wave (7) is described by the Bessel-Fubini expansion [19]. By performing similar calculations for the solution (8), we obtain an expression for its spectrum:

$$S(\omega) = \frac{1}{i\omega\left(\varepsilon/\rho c^3\right) z} \int_{-\infty}^{\infty} e^{-i\omega(T+\eta)} \left(e^{i\omega \frac{\varepsilon}{\rho c^3} zF} - 1 \right) dT .$$

Let's take into account that the variance of the value η is equal to $\left\langle \eta^2 \right\rangle = \frac{2}{c_0^2} \int_0^z (z-s) K_\varsigma(s) ds$, and at δ-correlation of fluctuations , $K_\varsigma(s) = D\delta(s)$ $\left\langle \eta^2 \right\rangle = \frac{Dz}{c_0^2}$. The mean value of $\langle \eta \rangle = 0$, if the mean value of the velocity fluctuations is zero. If the fluctuations ς are a Gaussian process, then η will also be a Gaussian process. Then we can write an expression for the characteristic function $\left\langle e^{-i\omega\eta}\right\rangle = e^{-\frac{\omega^2}{2}\left\langle\eta^2\right\rangle} = e^{-\frac{\omega^2}{2}\frac{Dz}{c_0^2}}$. Now the averaged spectrum is equal to:

$$\left\langle S(\omega)\right\rangle = \frac{1}{i\omega\left(\varepsilon/\rho c^3\right)z}\int_{-\infty}^{\infty} \mathrm{e}^{-i\omega T-\frac{\omega^2}{2}\left\langle\eta^2\right\rangle}\left(\mathrm{e}^{i\omega\frac{\varepsilon}{\rho c^3}zF}-1\right)dT\,. \tag{9}$$

Applying the inverse Fourier transform, we find the mean field:

$$\left\langle p\right\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\langle S(\omega)\right\rangle e^{i\omega\tau}d\omega = \int_{-\infty}^{\infty}\frac{1}{2\pi i\omega\left(\varepsilon/\rho c^3\right)z}e^{-\frac{\omega^2}{2}\left\langle\eta^2\right\rangle}\int_{-\infty}^{\infty}e^{i\omega(\tau-T)}\left(e^{i\omega\frac{\varepsilon}{\rho c^3}zF}-1\right)dTd\omega\,. \tag{10}$$

In (10) it is convenient to first calculate the derivative of the mean field

$$\frac{\partial\left\langle p\right\rangle}{\partial\tau} = \frac{1}{2\pi\left(\varepsilon/\rho c^3\right)z}\sqrt{\frac{2\pi}{\left\langle\eta^2\right\rangle}}\int_{-\infty}^{\infty}\left\{\exp\left(-\frac{1}{2\left\langle\eta^2\right\rangle}\left(\tau-T+\frac{\varepsilon}{\rho c^3}zF(T)\right)^2\right)-\exp\left(-\frac{(\tau-T)^2}{2\left\langle\eta^2\right\rangle}\right)\right\}dT\,. \tag{11}$$

For the N-wave (4), the solution (10) in dimensionless variables (3) has the form:

$$\begin{aligned}\frac{\left\langle p\right\rangle}{p_0} = \frac{\beta}{2x}\Bigg\{&\frac{1}{1+x}\Bigg[\frac{\theta+(1+x)}{\beta}\Phi\left(\frac{\theta+(1+x)}{\beta}\right)-\frac{\theta-(1+x)}{\beta}\Phi\left(\frac{\theta-(1+x)}{\beta}\right)+\\ &+\frac{1}{\sqrt{\pi}}\exp\left(-\frac{(\theta+(1+x))^2}{\beta^2}\right)-\frac{1}{\sqrt{\pi}}\exp\left(-\frac{(\theta-(1+x))^2}{\beta^2}\right)\Bigg]+\\ &+\frac{\theta-1}{\beta}\Phi\left(\frac{\theta-1}{\beta}\right)-\frac{\theta+1}{\beta}\Phi\left(\frac{\theta+1}{\beta}\right)+\frac{1}{\sqrt{\pi}}e^{-\frac{(\theta-1)^2}{\beta^2}}-\frac{1}{\sqrt{\pi}}e^{-\frac{(\theta+1)^2}{\beta^2}}\Bigg\},\end{aligned} \tag{12}$$

Where . $\beta=\beta(z)=\frac{\sqrt{2\left\langle\eta^2\right\rangle}}{\tau_0}=\frac{\sqrt{2Dz}}{c_0\tau_0}\equiv D_0\sqrt{x}$

However, the obtained solution (10) incorrectly describes the evolution of the N-wave (4). This can be easily verified by considering in (12) the limiting transition to the absence of fluctuations at : $\beta\to 0$ $(D\to 0)$

$$\frac{\left\langle p\right\rangle}{p_0} = \frac{1}{2x}\left\{\frac{1}{1+x}\left[|\theta+1+x|-|\theta-1-x|\right]+|\theta-1|-|\theta+1|\right\}. \tag{13}$$

The time profiles of the solution (11) are plotted in Fig. 2 for different distances. It can be seen that the obtained solution describes the blurring of the shock front, which does not correspond to the dynamics of the N-shaped shock pulse in a nonlinear medium. Thus, when averaging waves with discontinuities, it is necessary to obtain an explicit discontinuous solution for the profile beforehand.

EQUATION OF RUPTURE MOTION IN A MEDIUM WITH FLUCTUATIONS

Let us analyze the dynamics of the discontinuity motion in the wave described by equation (1). For the N-wave, the solution (8) can be written explicitly for the inverse function:

$$\tau = -\frac{\tau_0}{\mathrm{p}_0} p - \frac{\varepsilon}{\rho c^3} p z - \eta + \tau_0 .$$

Let us determine the position of the leading front during wave propagation. Let us introduce the following notations: p_1 is the minimum value of the pressure in the rupture, p_2 is the maximum pressure. Then for the leading front we can write:

$$p_1 = 0 , .\ \tau_2 = -\frac{\tau_0}{p_0} p_2 - \frac{\varepsilon}{\rho c^3} p_2 z - \eta + \tau_0 \qquad (14)$$

The equation of motion of the discontinuity coordinate τ_p can be obtained from the law of conservation of momentum:

$$\frac{d}{dz}\int_{p_1}^{p_2} \left(\tau(p) - \tau_p\right) dp = 0 , .\ \Rightarrow \int_{p_1}^{p_2} \frac{d\tau}{dz} dp = \left(p_2 - p_1\right)\frac{d\tau_p}{dz} \qquad (15)$$

We calculate: $\frac{d\tau}{dz} = -\frac{d\tau}{dp}\frac{dp}{dz} = -\frac{d\tau}{dp}\left(\frac{\varepsilon}{\rho c^3} p \frac{dp}{d\tau} + \varsigma \frac{dp}{d\tau}\right) = -\frac{\varepsilon}{\rho c^3} p - \varsigma$, and from (15) we obtain:

$$\frac{d\tau_p}{dz} = -\frac{\varepsilon}{2\rho c^3}(p_2 + p_1) - \varsigma . \tag{16}$$

Equations (14) and (16) fully describe the rupture motion. Solving them together, we obtain expressions for the amplitude and position of the rupture

:

$$p_2 = \frac{p_0}{\sqrt{1 + \frac{\varepsilon p_0}{\rho c^3 \tau_0}}}, . \ \tau_p = -\tau_0 \sqrt{1 + \frac{\varepsilon p_0}{\rho c^3 \tau_0}} - \eta + \tau_0$$

Finally, we obtain an explicit solution for the N-wave profile:

$$p = \begin{cases} -\frac{p_0}{\tau_0}\frac{\tau + \eta}{1 + x}, -T(x) - \eta < \tau < T(x) - \eta, \\ 0, \qquad -T(x) - \eta > \tau, \tau > T(x) - \eta, \end{cases} \tag{17}$$

where $T(x) = \tau_0 \sqrt{1 + x}$, distance *x is* defined in (3) Expression (17) allows us to correctly average the solution for a discontinuous wave. Using the spectral representation of the solution (17), we obtain an expression for the mean field:

$$\langle p \rangle = \frac{1}{2\pi} \int_{-T(z)}^{T(z)} p(\tau')d\tau' \int_{-\infty}^{\infty} e^{-\frac{\langle \eta^2 \rangle}{2}\omega^2 + i\omega(\tau - \tau')} d\omega = \frac{1}{\sqrt{2\pi\langle \eta^2 \rangle}} \int_{-T(z)}^{T(z)} p(\tau') \exp\left(-\frac{(\tau - \tau')^2}{2\langle \eta^2 \rangle}\right) d\tau' . \tag{18}$$

For the N-wave in dimensionless variables we finally obtain:

$$\frac{\langle p \rangle}{p_0} = \frac{1}{4}\frac{1}{1+x}\left[\left(\theta - 1 + \frac{D_0\sqrt{x}}{\sqrt{2}}\right)\left(\Phi\left(\frac{\theta - 1}{D_0\sqrt{x}}\right) - \Phi\left(\frac{\theta + \sqrt{1+x}}{D_0\sqrt{x}}\right)\right) + \right.$$
$$\left. + \frac{D_0\sqrt{x}}{\sqrt{\pi}}\left(\exp\left(-\frac{(\theta - 1)^2}{D_0^2 x}\right) - \exp\left(-\frac{\left(\theta + \sqrt{1+x}\right)^2}{D_0^2 x}\right)\right)\right]. \tag{19}$$

At $D_0 \to 0$ we obtain the following solution:

$$\frac{\langle p\rangle}{p_0} = -\frac{1}{2}\frac{\theta}{1+x}\left[\mathrm{sgn}\left(\theta+\sqrt{1+x}\right)-\mathrm{sgn}\left(\theta-\sqrt{1+x}\right)\right]. \qquad (20)$$

Expression (20) correctly describes the N-wave evolution in a homogeneous nonlinear medium.

Thus, the presence of a discontinuity in the wave profile must be taken into account before the averaging procedure, despite the fact that it itself introduces turbulent attenuation and smooths the shock fronts. However, this smoothing does not take into account the broadening of the pulse duration due to nonlinear effects, and leads only to blurring of the shock front in the region of its initial position. In fact, there is a competition of two processes - nonlinear broadening and turbulent damping.

EVOLUTION OF THE INITIAL TRIANGULAR PULSE

It is also interesting to trace the dynamics of the initial triangular pulse, in which the gap is not yet present:

$$F(\tau)=\begin{cases} p_0(\tau+\tau_0)/\tau_0, -\tau_0 \le \tau \le 0, \\ p_0(-\tau+\tau_0)/\tau_0, 0 < \tau \le \tau_0, \\ 0, |\tau| > \tau_0. \end{cases} \qquad (21)$$

Let us use formulas (10) and (11) obtained on the basis of averaging the spectral representation. Substituting in them the profile (21), we find:

$$\frac{\langle p\rangle}{p_0}=\frac{1}{2x}\left\{-D_0\sqrt{\frac{x}{\pi}}\exp\left(-\frac{(\theta+1)^2}{D_0^2x}\right)-(\theta+1)\Phi\left(\frac{\theta+1}{D_0\sqrt{x}}\right)+D_0\sqrt{\frac{x}{\pi}}\exp\left(-\frac{(\theta-1)^2}{D_0^2x}\right)+\right.$$
$$+(\theta-1)\Phi\left(\frac{\theta-1}{D_0\sqrt{x}}\right)+\left[\frac{1}{x-1}+\frac{1}{x+1}\right]\left[D_0\sqrt{\frac{x}{\pi}}\exp\left(-\frac{(\theta+x)^2}{D_0^2x}\right)+(\theta+x)\Phi\left(\frac{\theta+x}{D_0\sqrt{x}}\right)\right]+$$
$$+\frac{1}{x-1}\left[(\theta+1)\Phi\left(\frac{\theta+1}{D_0\sqrt{x}}\right)-D_0\sqrt{\frac{x}{\pi}}\exp\left(-\frac{(\theta+1)^2}{D_0^2x}\right)\right]+$$
$$\left.+\frac{1}{x+1}\left[(\theta-1)\Phi\left(\frac{\theta-1}{D_0\sqrt{x}}\right)-D_0\sqrt{\frac{x}{\pi}}\exp\left(-\frac{(\theta-1)^2}{D_0^2x}\right)\right]\right\}. \tag{22}$$

The profile of the solution (22) calculated at vanishing viscosity has the form:

$$\frac{\langle p\rangle}{p_0}=\frac{1}{2x}\left\{(\theta-1)\operatorname{sgn}(\theta-1)-(\theta+1)\operatorname{sgn}(\theta+1)+\frac{(x+\theta)\operatorname{sgn}(x+\theta)-(\theta+1)\operatorname{sgn}(\theta+1)}{x-1}+\right.$$
$$\left.+\frac{(x+\theta)\operatorname{sgn}(x+\theta)-(\theta-1)\operatorname{sgn}(\theta-1)}{x+1}\right\}. \tag{23}$$

The profile (23) is shown in Fig. 3 for different distances. As can be seen, at distances before the rupture formation, the profile distortion corresponds to the laws of nonlinear acoustics (curves 1-3). After the rupture formation, the profile calculation based on the spectral representation incorrectly describes the dynamics of the shock front - it blurs instead of shifting.

The averaged profiles for the triangular pulse after rupture formation based on the expression for the properly averaged field are shown in Fig. 4. Here one should also pay attention to the shape of the pulse. At a relatively small dispersion of fluctuations (Fig. 4a), the pulse has a characteristic shape with a twist and a clearly visible shock front, corresponding to its broadening due to nonlinear effects. The smoothing effect of turbulent attenuation is superimposed on this shape. Thus, we indeed obtain averaged discontinuity wave profiles. As the dispersion increases (Fig. 4b), this torsion disappears and the profile appears smoothed. If we return to Fig. 1 for the profiles obtained by the mean-field method, we can see that the twisting at the shock fronts is weakly pronounced at comparable values of dispersion to the plots in Fig. 4a.

Thus, we can conclude that the mean-field method does not accurately describe the most essential part of the averaged profile - the shock front and the degree of its steepness, underestimating these values. Thus, estimates based on this method may underestimate the expected acoustic fields in the turbulent atmosphere, which may have a negative impact on the environment.

CONCLUSION

Thus, the methods of obtaining closed equations for mean fields of acoustic waves in randomly inhomogeneous media and the results of calculations for wave profiles with discontinuities are considered. It is shown that the mean-field method does not accurately describe the transformation of the shock front under conditions of strong nonlinearity. At the same time, averaging of the exact dynamic solution also requires accuracy; first it is necessary to determine the position of the discontinuity in the profile.

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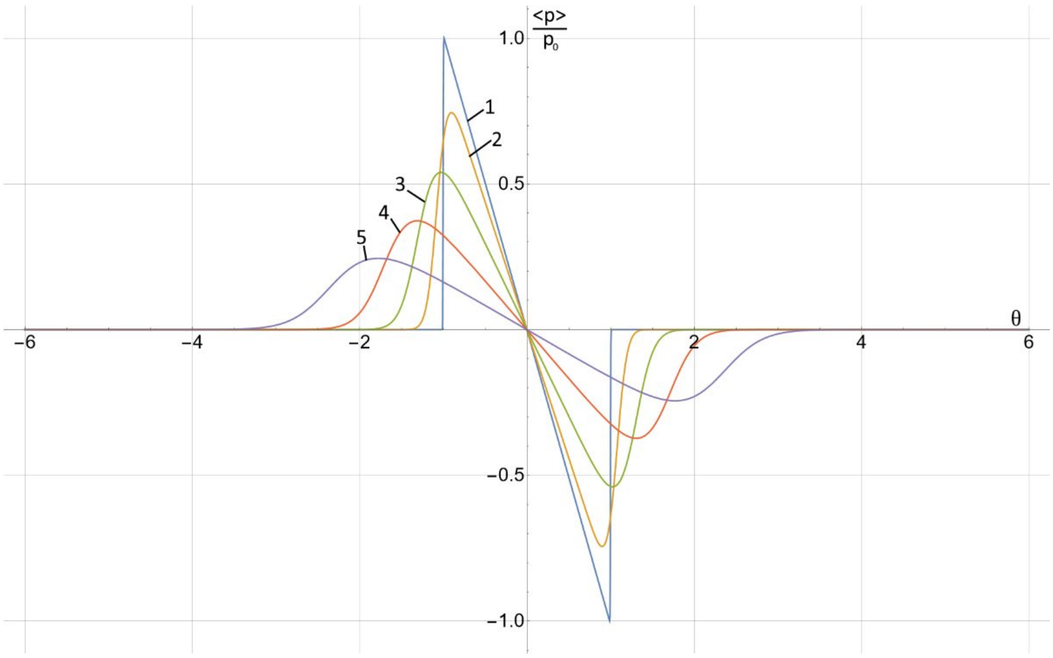
FIGURE CAPTIONS

Figure 1. Time profiles of mean pressure obtained by the mean-field method for the phase dispersion value $\Gamma = 0.05$ at distances x = 0.0001, 0.15, 0.7, 2, 5 (curves 1-5).

Fig. 2. Limit profiles at vanishingly small fluctuations of the medium obtained by averaging the spectral decomposition. Curves 1-6 correspond to distances x = 0.001, 0.3, 0.54, 0.8, 1.2, 2.

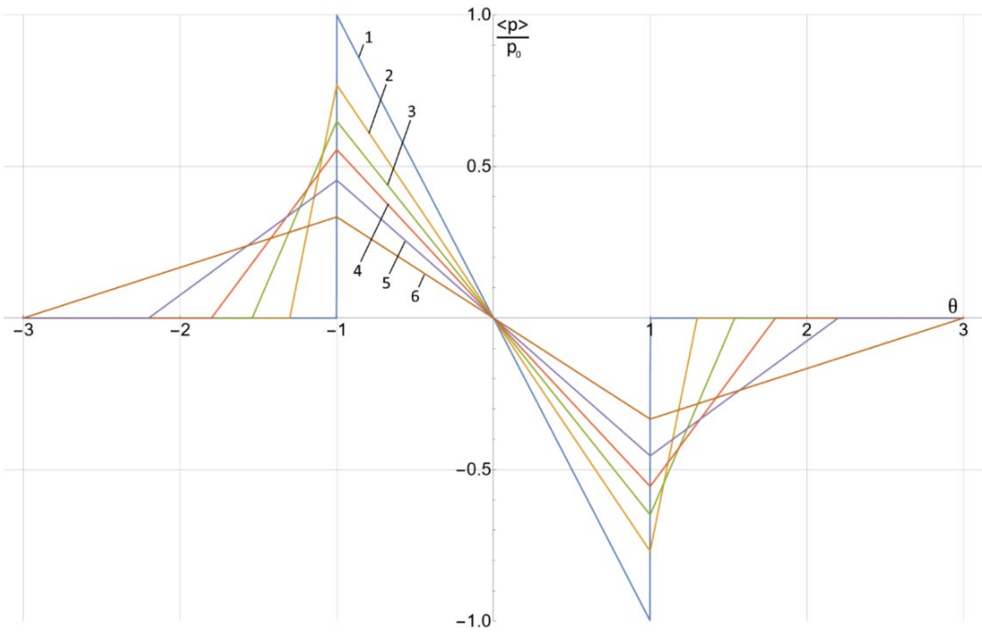
Fig. 3. Limit profiles of the initial triangular pulse at vanishingly small fluctuations of the medium obtained by averaging the spectral decomposition for distances x = 0.001, 0.3, 0.54, 0.8, 1.2, 2 (curves 1-6).

Fig. 4. Averaged profiles of the initial triangular pulse for fluctuation dispersion values $D_0 = 0.1$ (a), 0.5 (b) at distances x = 0.1, 0.8, 2, 5, 10 (curves 1-5).



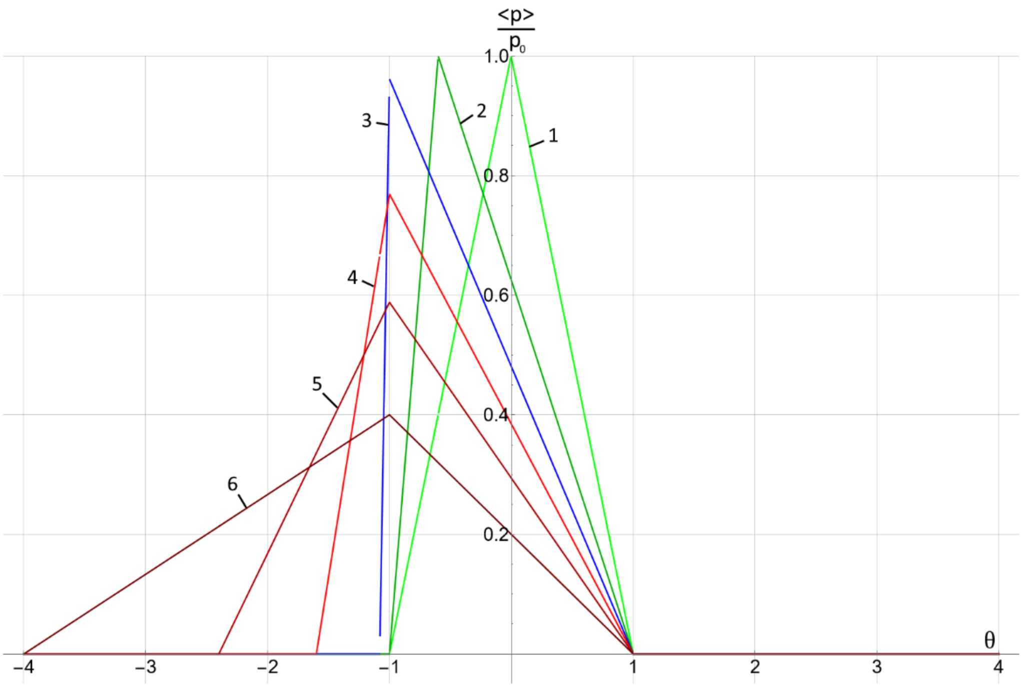
$\frac{<p>}{p_0}$
1.0
0.5
−0.5
−1.0
θ
−6
−4
−2
2
4
6
1
2
3
4
5

Fig. 1



$\frac{<p>}{p_0}$
1.0
0.5
−0.5
−1.0
θ
−3
−2
−1
1
2
3
1
2
3
4
5
6

Fig. 2



<p>
p_0
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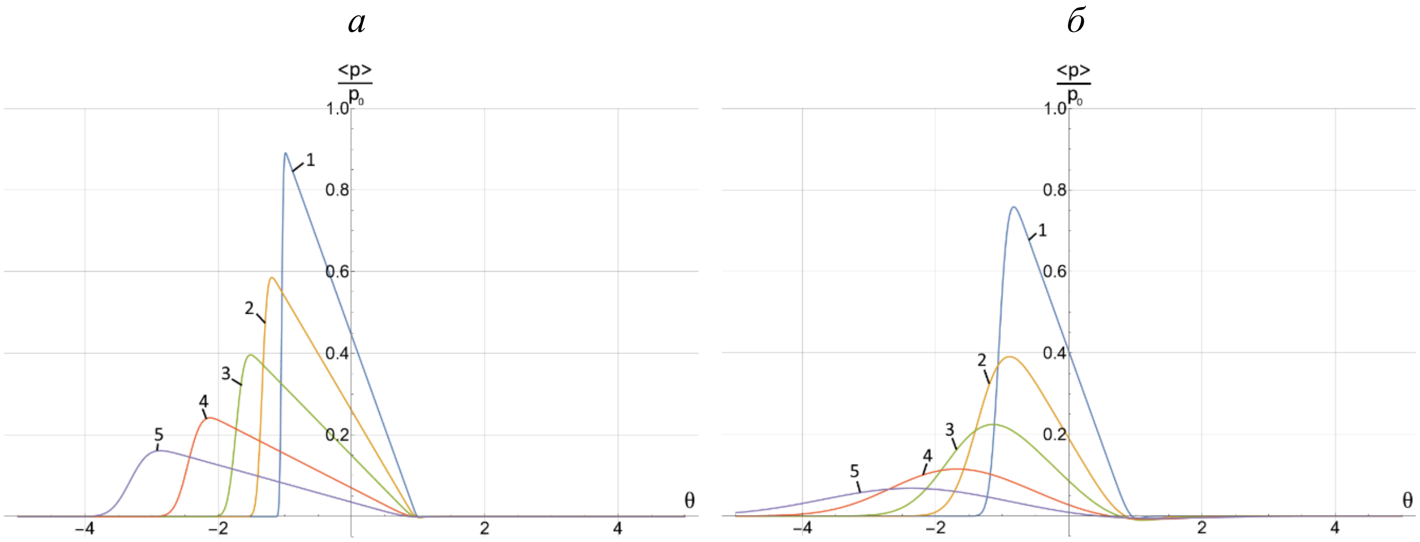
0.8
0.6
0.4
0.2
θ
−4
−3
−2
−1
1
2
3
4
1
2
3
4
5
6

Fig. 3



а
б
<p>
p_0
1.0
0.8
0.6
0.4
0.2
θ
−4
−2
2
4
1
2
3
4
5

Fig. 4